Leveraging Network Effects for Connected Products: Strategies and Implications for the Value Chain*

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Abstract

We explore how value chain participants can leverage the potential business value brought by the network effects of connected products with marginal production costs. We examine a classic two-tier value chain of a connected product that exhibits network effects. First, we present a benchmark model of a decentralized value chain operated through the wholesale price contract and demonstrate how the presence of network effects improves profitability and efficiency. Next, we explore two representative strategies value chain participants can employ to further exploit network effects: network expansion and engineering network strength. We find that these two strategies have a complementary relationship as the marginal production cost of the product declines. Finally, we investigate the impact of adopting these strategies on the efficiency loss of the decentralized value chain compared to the centralized one. The results suggest that both strategies could either increase or decrease such efficiency loss, and a decentralized value chain might bring about larger social welfare under network effects.

Keywords: Connected Product, Value Chain, Pricing, Engineering Network Effects

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1 Introduction

Nowadays, an increasing number of hardware devices are equipped with connectivity features, thus becoming connected products that exhibit significant network effects (Porter and Heppelmann 2014, Tien 2015).[1] For instance, Apple's AirTag is designed to be attached to items; if these items are ever lost or stolen, their geographic data can be transmitted to Apple devices nearby through encrypted connections to help locate them.[2] The literature indicates that the widespread use of connected products generally aligns with Metcalfe's law (i.e., the value of a network is proportional to the square of the number of its nodes, Metcalfe 2013), generating network value (Adner et al. 2019) and transforming businesses (Siebel 2019, p.109). As a result, business strategies for these connected products must adapt to the proliferation of the network effects.

To monetize network effects, conventional wisdom from software literature suggests two approaches. The first is the "network expansion strategy," which entails adding more network nodes to generate stronger network effects due to Metcalfe's law (e.g., Haenlein and Libai 2017, Gelper et al. 2021). The second is the "connection strengthening strategy," which seeks to engineer the strength of network connection to increase its benefit for each participant (e.g., Aral and Walker 2011, Dou et al. 2013). However, the applicability of these strategies from the software industry to connected devices remains unclear. First,

[1] The worldwide number of connected devices is projected to increase to 43 billion by 2023, an almost threefold increase from 2018. See the following link for more details. https://www.ericsson.com/en/press-releases/2019/6/ericsson-mobility-report-5g-uptake-even-faster-than-expected.

[2] https://www.wired.com/story/apple-find-my-cryptography-bluetooth/

the marginal production cost of such products is significant, making the low-price or even free strategies for expanding user networks potentially prohibitive. Second, unlike software, connected products are physical devices that cannot be downloaded or transmitted via the Internet. They must be delivered through the value chain, as evidenced when Google sells its smart home devices via Best Buy and Apple sells AirTag through Amazon. Given this, the interactions between value chain participants will inevitably influence the business value of network effects. This raises the central questions of our study: How can value chain participants of connected products harness the potential value of network effects? And, how do network effects affect value chain profits and social welfare?

To address these questions, this paper delves into a two-tier value chain of a connected product that exhibits network effects. We start with two benchmark findings from a baseline model employing the wholesale price contract: First, as network effects of the product increases, value chain participants are better off. Second, the efficiency loss caused by the double marginalization can be eliminated, yet this requires a very strong network effect. We then explore how value chain participants can take full advantage of network effects. Specifically, we examine how the value chain can use the two aforementioned strategies: the network expansion strategy (illustrated by seeding) and the connection strengthening strategy (represented by investments in engineering network effects).

Our model provides several insights regarding the optimal use of these two strategies. First, when aiming to expand the network through seeding, it's crucial for value chain participants to precisely identify customer types and target them accurately. If not, the benefits from the seeding strategy might not offset its associated marginal production costs. We demonstrate that either the manufacturer or the retailer may seed the market in equilibrium, but not both simultaneously. In such scenarios, the seeding strategy can effectively lower the threshold for the value chain to achieve efficiency. Second, in terms of strengthening the connection, we examine when and how much to invest in network features to enhance connection strength. Our findings suggest that the optimal investment might significantly increase if the efficiency of the investment is high and the marginal cost of product production is low. In essence, production efficiency could trigger a radical investment into achieving a "disruptive" level of network effects.

We then introduce a generalized setting that simultaneously encompasses the two strategies mentioned above, and demonstrate that all the previous results remain robust. Furthermore, we identify a complementary relationship between these two strategies as the marginal production cost decreases. To put it differently, if distributing the connected product for free as a seed remains prohibitively expensive, investing heavily in engineering network effects might not be optimal either. We also delve into the impact of these strategies on the efficiency of the decentralized value chain. Our findings indicate that both strategies could either improve or diminish value chain efficiency. Moreover, while customer surplus increases with network scale, it isn't maximized under the central planner's profit-maximizing strategy. In essence, the central planner under-invests due to the positive externalities benefiting customers. Conversely, it's plausible that the decentralized value chain, with significant investment in engineering network effects, might yield higher social welfare than an integrated value chain. Such findings present a novel contribution to the literature, shedding light on the nuanced impacts of network effects on value chain management.

2 Literature Review

This paper intersects with three streams of prior literature. The first stream delves into the pricing challenges under network effects, a topic widely examined since the seminal works by Rohlfs (1974) and Varian (2001). When customers are interconnected, an externality is created, often prompting price discrimination (e.g., Jing 2007, Chien and Chu 2008). The early adopters bring added value to subsequent users due to Metcalfe's law (Metcalfe 2013). This dynamic fosters various freemium strategies, aiming to expand network size using free products and subsequently capitalize on the generated externality (Jiang and Sarkar 2009, Cheng and Tang 2010). Our paper explores the seeding strategy (Libai et al. 2005) – where a segment of the potential market accesses the product for free. From a value chain perspective, we derive several insights on seeding under network effects. Notably, either the upstream manufacturer or the downstream retailer may deploy seeding in equilibrium, but not both. While our model resembles the two-tier structure with network effects in Yi et al. (2019), their focus is primarily on software, sidelining the marginal cost. In contrast, we emphasize the significance of the marginal production cost, contending that radical investment in enhancing network effects isn't optimal if the product remains expensive to manufacture.

The second stream focuses on overcoming the double marginalization effects. We introduce leveraging network effects as a novel solution. Acknowledging the implications of diverse technologies, existing literature has explored the repercussions of double marginalization in varied settings such as performance-based advertising (Dellarocas 2012), word-of-mouth effects (Han et al. 2021), and software-as-a-service (Demirkan et al. 2010), among others. Starting with a classic two-tier value chain using the wholesale price contract, our paper identifies that network effects could potentially offset the efficiency loss inherent in such contracts. Furthermore, our generalized model demonstrates that double marginalization can spur upstream product innovation regarding network effects, influencing the downstream business model (i.e., seeding). Intriguingly, this suggests that double marginalization might inadvertently produce favorable outcomes as a product's network effects intensify.

Lastly, the third body of literature delves into managing connected products. Although leveraging data for personalized recommendations and pricing optimizations is prevalent (Varian 2019), the ripple effects on product development and value chain performance when products are increasingly interconnected remain underexplored (Hagiu and Wright 2020). Adner et al. (2019) observed that firm boundaries have become increasingly porous, prompting companies to venture into their value-creation partners' domains. They propose a possible "connectivity revolution," aligning with our findings on the strategy of engineering network effects. Significantly, our paper posits that this revolution can be spurred by double marginalization effects, leading to noteworthy repercussions for both value chain profits and societal welfare. Another relevant study is Sun and Ji (2022), which probes the ramifications of Internet of Things (IoT) technology in a channel setting. Their study, however, doesn't incorporate network effects because the value derived from IoT doesn't rely on user base size. In contrast, our analytical framework comprehensively integrates network effects in both its exogenous and endogenous forms.

3 The Baseline Model

The Value Chain. Consider a classic value chain with two participants: An upstream manufacturer (labeled as M, such as Google's smart home device manufacturer) produces a product and sells it through a downstream retailer (labeled as R, such as Best Buy). Each customer demands at most one unit of the product and derives a standalone value (or "type"), θ, which measures the value of using the product independently from others (e.g., playing music from the smart home devices). We assume that θ is uniformly distributed on the interval $[0, 1]$. In addition to the standalone value, users also receive a greater network-based value if there are a larger number of peer users. For example, users can benefit when Google uses the data contributed from these devices to improve their performance (e.g., the accuracy of voice recognition improves when data from various accents are available). Denote such network-based benefit as bN where b is the strength of the network effect and N is the number of users. Hence, the utility of a type-θ user is given by:

$$U(\theta; N) := \theta + bN - p, \tag{1}$$

in which p is the market price of the product. Two remarks pertain to Equation (1). First, the additive form of utility function indicates the separability between the standalone value (θ) and network effects (bN). Specifically, for the connected product examined in our paper, the network effects increase with both the connectivity (b) and the network size (N). Second, Equation (1) implies that the network-based benefits are homogeneous among users, which aligns with prior literature studying network effects (e.g., Katz and Shapiro 1985, Dou et al. 2017). [3]

[3] In Appendix E, we consider the multiplicative form of the utility function wherein the network bene-

Customer Segmentation. We assume that $b \in (0,1)$,[4] and that every customer has a zero-utility outside option. This implies the existence of marginal customers with a cutoff type $\hat{\theta} \in [0,1]$ who receive a zero utility.[5] Therefore, the equilibrium price is $p = \hat{\theta} + b(1 - \hat{\theta})$. With this one-to-one mapping, optimizing p is equivalent to determining the optimal $\hat{\theta}$.

Value Chain Participants' Problems. The manufacturer first chooses the wholesale price, w. The marginal cost of producing the product is $c \in (0,1)$. Subsequently, the retailer purchases from the manufacturer and determines the market price, p. We refer to this game as the *decentralized case*. By backward induction, given a particular w, the retailer's profit is as follows:

$$\Pi_R(\hat{\theta}; w) \coloneqq (p - w)N = [\hat{\theta} + b(1 - \hat{\theta}) - w](1 - \hat{\theta}). \tag{2}$$

Solving the retailer's best response, $\hat{\theta}(w)$, we can then deduce the manufacturer's decision by choosing the optimal wholesale price w from the profit function below:

$$\Pi_M(w) \coloneqq (w - c)N = (w - c)[1 - \hat{\theta}(w)]. \tag{3}$$

A subgame-perfect Nash equilibrium (hereafter referred to as *equilibrium* for brevity), $\{w^*, \theta^*(w)\}$, ensures that each value chain participant maximizes its own profit considering the other participant's strategy.

For comparison with the decentralized case, we introduce the *centralized* case, wherein the manufacturer and the retailer collaborate as a unified entity (called the "central planner" and labeled by I) to maximize the combined profit (or "industry profit").

fits are associated with each user's standalone value (i.e., $U = \theta(1 + bN) - p$). The results suggest our key insights remain valid under this multiplicative form. We thank an anonymous reviewer for this suggestion.

[4]From our subsequent results, if $b \geq 1$, the market is always fully covered in equilibrium.

[5]Given that a customer's utility increases with her type, if a certain type $\theta \in (0,1)$ prefers to buy the product, all higher types will also choose to do so. Consequently, if the cutoff customer's type is $\hat{\theta}$, the equilibrium number of users is $N = 1 - \hat{\theta}$ (i.e., the total number of customers with a type exceeding $\hat{\theta}$).

$$\Pi_I := (p_I - c)N = [\hat{\theta}_I + b(1-\hat{\theta}_I) - c](1-\hat{\theta}_I). \tag{4}$$

The equilibrium for each of the two cases described above is detailed in Lemma 1.[6]

Lemma 1. *There exists a unique equilibrium in each case such that:*

i. (Centralized case) The central planner's strategy and the corresponding profit are

$$\{\hat{\theta}_I^*, \Pi_I^*\} = \begin{cases} \{\frac{1+c-2b}{2(1-b)}, \frac{(1-c)^2}{4(1-b)}\} & 0 < b \leq \frac{1+c}{2}; \\ \{0, b-c\} & \frac{1+c}{2} < b < 1. \end{cases}$$

ii. (Decentralized case) The on-path strategies and the value chain participants' profits are

$$\{w^*, \hat{\theta}_D^*, \Pi_M^*, \Pi_R^*\} = \begin{cases} \{\frac{1+c}{2}, \frac{3+c-4b}{4(1-b)}, \frac{(1-c)^2}{8(1-b)}, \frac{(1-c)^2}{16(1-b)}\} & 0 < b \leq \frac{3+c}{4}; \\ \{2b-1, 0, 2b-1-c, 1-b\} & \frac{3+c}{4} < b < 1. \end{cases}$$

The novelty of our model lies in embracing the impacts of network-based benefits in the value chain, as captured by b. Lemma 1 demonstrates that both the central planner's profit (Π_I^*) and the industry profit in the decentralized case ($\Pi_M^* + \Pi_R^*$) increase with b, highlighting the favorable role of network effects. More interestingly, as b increases, the cutoff points, $\hat{\theta}_D$ and $\hat{\theta}_I$, approach zero. This suggests that the value chain, whether centralized or decentralized, aims to expand the user base as much as possible for profit generation. With very strong network effects (i.e., $b > \frac{3+c}{4}$), the profit in both cases converges (i.e., $\Pi_M^* + \Pi_R^* = \Pi_I^*$) as they both cover the market at the same price.

These findings about user base expansion and profit convergence prompt us to delve deeper into value chain efficiency. Established literature confirms that in the absence of

[6]All notation is summarized in Appendix A. Specifically, an asterisk denotes an equilibrium variable.

network effects (i.e., $b = 0$), the retailer's margin is smaller than that of a central planner. This disparity results from the upward distortion in pricing – known as double marginalization, and leads to a reduced industry profit for the value chain (i.e., efficiency loss). Given this, how does the presence of network effects affect the role of double marginalization?

To address this question, we begin by establishing a measure for efficiency loss. Since incremental network effects enhance the potential overall profit, we define the efficiency loss ratio L by dividing the profit gap by the centralized profit:

$$L = \frac{\Pi_I^* - (\Pi_M^* + \Pi_R^*)}{\Pi_I^*} = 1 - \frac{\Pi_M^* + \Pi_R^*}{\Pi_I^*}.$$

Furthermore, the social welfare can be expressed as $W = \int_{\underline{\theta}}^1 [\theta + b(1-\underline{\theta}) - c]\, d\theta$. Corollary 1 presents the results for efficiency loss and social welfare.

Corollary 1. *The equilibria of the centralized and decentralized cases are such that:*

i. (Efficiency loss) The efficiency loss ratio is given by

$$L = \begin{cases} \frac{1}{4} & 0 < b \leq \frac{1+c}{2}; \\ 1 - \frac{3(1-c)^2}{16(1-b)(b-c)} & \frac{1+c}{2} < b \leq \frac{3+c}{4}; \\ 0 & \frac{3+c}{4} < b < 1. \end{cases}$$

ii. (Social welfare) The social welfare is given by

$$W_I = \begin{cases} \frac{(1-c)^2(3-2b)}{8(1-b)^2} & 0 < b \leq \frac{1+c}{2}; \\ \frac{1}{2} + b - c & \frac{1+c}{2} < b \leq 1; \end{cases} \quad W_D = \begin{cases} \frac{(1-c)^2(7-6b)}{32(1-b)^2} & 0 < b \leq \frac{3+c}{4}; \\ \frac{1}{2} + b - c & \frac{3+c}{4} < b < 1. \end{cases}$$

Figure 1 illustrates the various impacts of network effects as described in Lemma 1 and

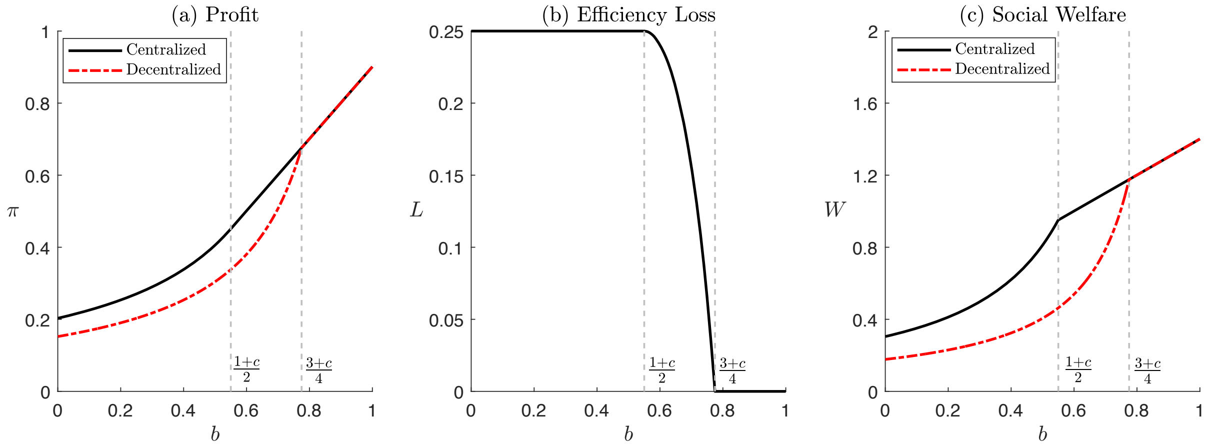

Figure 1: Profits and Efficiency Loss in the Value Chain

Corollary 1. This also serves as a benchmark for our subsequent analysis. A key takeaway is that efficiency loss can be eliminated if the product exhibits network effects (and the social welfare also converges) – excellent news for the industry, but only under a considerably large b. Essentially, as network effects emerge, the operational objectives of value chain participants align more closely: they aim to benefit from network effects by expanding the user base. Prioritizing the acquisition of more customers at a reduced price counteracts the upward price distortion observed in traditional settings without network effects.

While network effects can benefit the value chain, their full potential is only realized when b is notably large. Can the value chain participants exploit network effects more proactively rather than merely waiting for b to strengthen? The literature suggests two approaches. The first is to increase the number of new "nodes" in the network, thereby enhancing the connections for existing nodes and generating more network effects following Metcalfe's law (e.g., acquiring more users as a larger network values more). The second is to intensify the strength of each "edge" in the network, deriving greater value from each connection (e.g., offering more connectivity-based features). In this paper, we delve into both strategies. The first, termed "seeding," involves providing the product for free to users who wouldn't otherwise purchase it (aligning with the first approach mentioned). The second strategy focuses on investing in connection strength (corresponding to the second approach). Through this framework, our goal is to comprehensively understand how to enhance the network effects' value for the value chain.

4 Strategies for Leveraging Network Effects

Building on our baseline model in Section 3, this section delves into two approaches to capitalize on network effects within the value chain.

4.1 Expanding the Network via Seeding

The first approach contemplates the network-expansion method wherein both the manufacturer and the retailer proactively give products away for free to bolster the user base. This is referred to as the "seeding" strategy. Through seeding, the value chain can achieve a "critical mass" for subsequent purchases, as the network benefits are amplified by the customers who receive the free product (i.e., "seeded" customers). For example, Amazon UK gives away Apple's AirTag for free[7] and Apple also gave 20,000 AirTag away in Japan in 2021.[8] Morgan Stanley also suggested that Google distribute free home smart speakers to grow the network.[9]

The seeding strategy is built on two main insights. First, a customer should not be seeded if she is likely to make a purchase anyway. Therefore, to maximize profits in con-

[7] https://www.manchestereveningnews.co.uk/whats-on/shopping/shoppers-can-free-apple-airtag-25602619

[8] https://www.theapplepost.com/2021/12/27/apple-giving-away-free-limited-edition-airtag-with-select-iphone-purchases-as-part-of-new-japanese-new-year-promo/

[9] https://www.cnbc.com/2018/06/28/morgan-stanley-google-should-give-out-free-smart-speakers-to-beat-ama.html

texts like a fully connected network topology (e.g., the Metcalfe network), it is optimal to target customers with the lowest types (i.e., a small θ) for seeding. Second, the benefits of seeding diminish if targeting is costly. To demonstrate the full potential of seeding, we consider the optimal scenario where the value chain, whether centralized or decentralized, can target the lowest-type customers for seeding without additional costs.[10]

First, consider the centralized case. The central planner's strategy comprises the seeding level α_I and the cutoff point $\hat{\theta}_I$. The central planner's profit is given by:

$$\Pi_I := [\hat{\theta}_I + (1 - \hat{\theta}_I + \alpha_I)b - c](1 - \hat{\theta}_I) - c\alpha_I. \tag{5}$$

Note that $\hat{\theta}_I \in [\alpha_I, 1]$ must hold because all seeds are targeted at customers in the interval $[0, \alpha_I]$. Compared to Equation (4), introducing α_I leads to two changes. On the one hand, the equilibrium number of users becomes $1 - \hat{\theta}_I + \alpha_I$, encompassing both the paying customers, $1 - \hat{\theta}_I$, and the seeded customers, α_I. On the other hand, distributing α_I units of the product for free incurs a marginal cost of $c\alpha_I$. The subsequent lemma characterizes the central planner's optimal strategy and the associated profit.

Lemma 2. *With seeding, the central planner's optimal strategy is such that:*

i. When $c < c^I$, $\alpha_I^* = \frac{1-b}{2}$, $\hat{\theta}_I^* = \frac{1-b}{2}$, resulting in $\Pi_I^* = \frac{(1+b)^2}{4} - c$;

ii. When $c \geq c^I$, $\alpha_I^* = 0$, $\hat{\theta}_I^* = \frac{1+c-2b}{2(1-b)}$, resulting in $\Pi_I^* = \frac{(1-c)^2}{4(1-b)}$,

where the cutoff is $c^I := 1 - 2(1-b) + (1-b)^{\frac{3}{2}} \in (0, 1)$.

Lemma 2 suggests that seeding is profitable (i.e., $\alpha_I^* > 0$) for the central planner if and only if c is below the threshold c^I. Moreover, the threshold, c^I, increases with b, indicating

[10]Consistent with our intuition, we show in Appendix D that seeding doesn't improve profits when the value chain cannot target specific customers. Appendix C examines situations where targeting comes with additional costs beyond the product's marginal cost. We thank the anonymous reviewers for suggesting these perspectives to highlight the limitations of the seeding strategy.

that seeding becomes more appealing to the central planner as b rises. In stark contrast to the baseline model without seeding, the market will be fully covered for any b so long as $c < c^I$ (we show later that this is similarly true for the decentralized case). Thus, seeding acts as a radical and less costly way to expand the consumer network.

Next, we consider the decentralized case where the manufacturer and the retailer make their strategy decisions sequentially and independently. The manufacturer first chooses a wholesale price w and a seeding volume $\alpha_M \in [0, 1]$; the retailer follows to choose a retail price p and a seeding volume $\alpha_R \in [0, 1 - \alpha_M]$.[11] Using backward induction, given the manufacturer's strategy (α_M, w), the retailer maximizes the following profit

$$\Pi_R(\alpha_R, \hat{\theta}_D; \alpha_M, w) \coloneqq [(1 - b)\hat{\theta}_D + (1 + \alpha_M + \alpha_R)b - w](1 - \hat{\theta}_D) - w\alpha_R. \tag{6}$$

Given the retailer's best responses, the manufacturer chooses the wholesale price w and the seeding volume α_M to maximize the profit

$$\Pi_M(\alpha_M, w) \coloneqq (w - c)(1 - \hat{\theta}_D + \alpha_R) - c\alpha_M. \tag{7}$$

Lemma 3 below presents the equilibrium outcome.

Lemma 3. *With seeding, the equilibrium of the decentralized case is such that:*

i. When $b > \hat{b}$ and $c < c^R$, $\alpha_M^* = 0$, $w^* = c^I$, $\alpha_R^* = \frac{1-b}{2}$, $\hat{\theta}_D^* = \frac{1-b}{2}$, resulting in $\Pi_M^* = c^I - c$ and $\Pi_R^* = \frac{(1+b)^2}{4} - c^I$;

ii. When $b \leq \hat{b}$ and $c < c^M$, $\alpha_M^* = \frac{3(1-b)}{2(2-b)}$, $w^* = \frac{1+b}{2}$, $\alpha_R^* = 0$, $\hat{\theta}_D^* = \frac{3(1-b)}{2(2-b)}$, resulting in $\Pi_M^* = \frac{(1+b)^2}{4(2-b)} - c$ and $\Pi_R^* = \frac{(1-b)(1+b)^2}{4(2-b)^2}$;

iii. When $c \geq \max\{c^R, c^M\}$, $\alpha_M^* = \alpha_R^* = 0$, $w^* = \frac{1+c}{2}$, $\hat{\theta}_D^* = \frac{3+c-4b}{4(1-b)}$, resulting in $\Pi_M^* = \frac{(1-c)^2}{8(1-b)}$ and $\Pi_R^* = \frac{(1-c)^2}{16(1-b)}$,

[11] That is, we assume that the manufacturer and the retailer can coordinate effectively to avoid double-seeding the same customer. This represents the ideal scenario for seeding, which we examine as the upper bound of seeding performance.

where the cutoffs are $\hat{b} := \frac{9\sqrt{17}-17}{32} \approx 0.63$, $c^R := \max\{0, 1-4(1-b)+2\sqrt{2}(1-b)^{\frac{5}{4}}\}$, *and* $c^M := 1-4(1-b)+3(1-b)\sqrt{\frac{2-2b}{2-b}} \in (0,1)$.

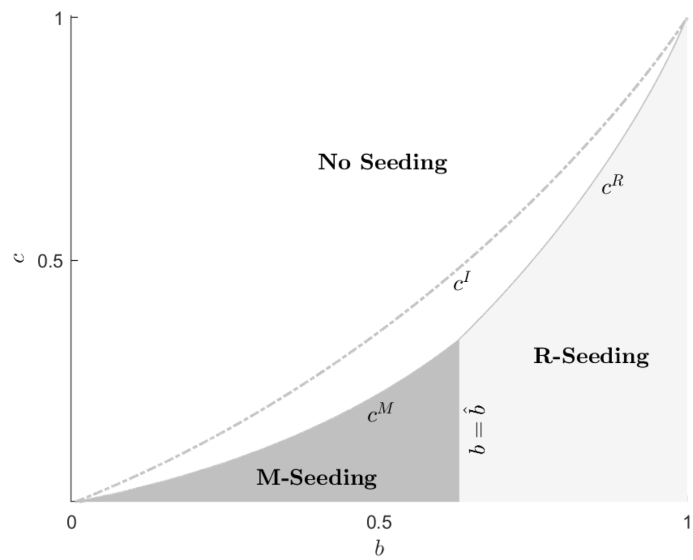

Figure 2: Equilibrium Outcomes with Seeding

Figure 2 visualizes when and how the value chain can (further) monetize network effects by employing seeding to enlarge the user base. When the network effect is relatively strong ($b > \hat{b}$), a manufacturer charges a low wholesale price if $c < c^R$, which allows the retailer to seed the customers (referred to as *R-seeding*). On the other hand, if both the network effect and the marginal cost are relatively low (i.e., $b \leq \hat{b}$ and $c < c^M$), the manufacturer prefers to seed the customers directly (referred to as *M-seeding*) and charge a higher wholesale price. Otherwise, if the marginal cost is sufficiently high ($c > \max\{c^R, c^M\}$), neither the manufacturer nor the retailer have any incentives to seed, in which case the equilibrium reduces to Lemma 1.

It should be highlighted that our analytical results suggest a very clear structure: It is never an equilibrium for the manufacturer and the retailer to seed the market simultaneously. The intuition is that whenever the retailer decides to seed, it seeds all the remaining market. This gives the manufacturer with an incentive to free-ride on the retailer's seeding.

As a result, the manufacturer faces a tradeoff when choosing its optimal strategy: it can either directly seed the market and charge a wholesale price $\frac{1+b}{2}$ or induce the retailer to seed, thus saving on the seeding cost by charging a lower price $c^I < \frac{1+b}{2}$ (i.e., reducing its margin). The difference $\frac{1+b}{2} - c^I$ decreases in b and vanishes as b approaches 1. Thus, the manufacturer prefers R-seeding to M-seeding if and only if b is sufficiently high.

4.2 Engineering Network Effects

The second approach is to strengthen network effects by investing in the connectivity features (i.e., engineering network effects). Specifically, either the central planner or the manufacturer can develop new connectivity features, ensuring that each node derives greater benefits from every connection.

Consider a cost function $g(b) = \gamma b^2$ where γ is the coefficient of marginal innovation cost for engineering network effects. We assume that $\gamma > \frac{1}{2}$ to ensure a reasonable level of network effect. Our results are valid for a broad class of the cost function $g(b)$. In line with the baseline model, we consider $b \in [0, \bar{b}]$ with $\bar{b} < 1$ and close to 1.

The timeline changes as follows. The producer (i.e., the central planner in the centralized case or the manufacturer in the decentralized case) first observes the cost function $g(b)$ and the marginal cost $c \in (0, 1)$; then, the producer chooses a strength of network effect $b \in [0, \bar{b}]$; finally, given b and c, the game proceeds as in the baseline model.

We follow a similar order to start with the centralized case. We can directly derive the central planner's profit (net from the investment) based on Lemma 1:

$$\Pi_I(b) := \begin{cases} \frac{(1-c)^2}{4(1-b)} - \gamma b^2 & 0<b \leq \frac{1+c}{2}; \\ b - c - \gamma b^2 & \frac{1+c}{2}<b \leq 1. \end{cases}$$

Although a closed-form solution isn't obtainable for all c, Lemma 4 still provides characterization of the optimal strategy and conditions for the central planner.

Lemma 4. *When the network effects can be engineered, the central planner's optimal strategy is as follows:*

i. When $c < c_\gamma^I$, $b_I^* = b_I^h := \frac{1}{2\gamma}$, leading to $\hat{\theta}_I^* = 0$ and $\Pi_I^*(b_h^I) = \frac{1}{4\gamma} - c$;

ii. When $c \geq c_\gamma^I$, $b_I^* = b_I^l \in (0, b_I^h)$, which is the minimal root of

$$b(1-b)^2 = \frac{(1-c)^2}{8\gamma},$$

with ${b_I^l}'(c) < 0$ and ${b_I^l}''(c) > 0$. Therefore, $\hat{\theta}_I^* = \frac{1-2b_I^l+c}{2(1-b_I^l)}$ and $\Pi_I^*(b_I^l) = \frac{(1-c)^2}{4(1-b_I^l)} - \gamma {b_I^l}^2$.

The cutoff c_γ^I is a nonincreasing function of γ. If $\gamma < \tilde{\gamma}^I$ for some $\tilde{\gamma}^I \in \left(\frac{8}{9}, 1\right)$, $c_\gamma^I > 0$ is the unique root of $\Pi_I^*(b_I^h) = \Pi_I^*(b_I^l)$, which decreases with γ. However, when $\gamma \geq \tilde{\gamma}^I$, $c_\gamma^I \equiv 0$ and the equilibrium is always given by item ii.

An implication drawn from Lemma 4 is that the central planner can cover the entire market only if $\gamma < \tilde{\gamma}^I$ and $c < c_\gamma^I$. Otherwise, employing a lower price to achieve full market coverage—even with the flexibility of investing in connection strength—is not the central planner's optimal strategy.

Next, we consider the decentralized case. The manufacturer's profit is given by

$$\Pi_M(b) := \begin{cases} \frac{(1-c)^2}{8(1-b)} - \gamma b^2 & 0<b \leq \frac{3+c}{4}; \\ 2b - c - 1 - \gamma b^2 & \frac{3+c}{4}<b \leq 1. \end{cases}$$

We denote the optimal decision in the decentralized value chain as b_D^*. Lemma 5 that follows characterizes the equilibrium.

Lemma 5. *When the network effect can be engineered, the equilibrium of the decentralized case is as follows:*

i. *When* $c < c_\gamma^D$, $b_D^* = b_D^h := \bar{b}$. *Consequently,* $w^* = 2b_D^h - 1$, $\hat{\theta}_D^* = 0$, $\Pi_M^*(b_D^h) = 2b_D^h - 1 - c - \gamma {b_D^h}^2$, *and* $\Pi_R^*(b_D^h) = 1 - b_D^h$.

ii. *When* $c \geq c_\gamma^D$, $b_D^* = b_D^l \in (0, b_D^h)$, *which is the minimal root of*

$$b(1-b)^2 = \frac{(1-c)^2}{16\gamma},$$

with ${b_D^l}'(c) < 0$ *and* ${b_D^l}''(c) > 0$. *Therefore,* $w^* = \frac{1+c}{2}$, $\hat{\theta}_D^* = \frac{3+c-4b_D^l}{4(1-b_D^l)}$, $\Pi_M^*(b_D^l) = \frac{(1-c)^2}{8(1-b_D^l)} - \gamma {b_D^l}^2$, *and* $\Pi_R^*(b_D^l) = \frac{(1-c)^2}{16(1-b_D^l)}$.

The cutoff c_γ^D *is a nonincreasing function of* γ. *If* $\gamma < \tilde{\gamma}^D$ *for some* $\tilde{\gamma}^D < \tilde{\gamma}^I$, $c_\gamma^D > 0$ *is the unique root of* $\Pi_M^*(b_D^h) = \Pi_I^*(b_D^l)$ *and decreases with* γ. *However, when* $\gamma \geq \tilde{\gamma}^D$, $c_\gamma^D \equiv 0$ *and the equilibrium is always given by item ii.*

Upon closer examination of Lemmas 4 and 5, we observe an intriguing insight: The optimal strength of network effects does not shift continuously with its associated marginal cost, γ. Instead, in either the centralized or decentralized scenario, it may become optimal to make a radical investment for a "disruptive" enhancement of connection strength. Corollary 2 below summarizes this feature.

Corollary 2. *When the network effects can be engineered, the following holds:*

i. b_I^* *decreases in* γ *and jumps downward at* $\gamma = {c_\gamma^I}^{-1}(c)$ *if* $c_\gamma^I > 0$.

ii. b_D^* *decreases in* γ *and jumps downward at* $\gamma = {c_\gamma^D}^{-1}(c)$ *if* $c_\gamma^D > 0$.

Based on Corollary 2, we compare the optimal network strength (or investment scales to engineering network effects) between the centralized and decentralized cases. The comparison reveals a counterintuitive finding, presented in Proposition 1.

Proposition 1. *When both the coefficient of marginal innovation cost and the marginal production cost are sufficiently small (i.e., $\gamma < \tilde{\gamma}^D$ and $c < \min\{c_\gamma^I, c_\gamma^D\}$), the equilibrium level of investment in engineering network effects in the decentralized value chain is higher than the central planner's choice (i.e., $b_D^* > b_I^*$). If $\gamma \geq \tilde{\gamma}^D$, then $b_I^* > b_D^*$.*

The results of Proposition 1 differ significantly from traditional value chain research. The conventional wisdom is that due to double marginalization, the marginal returns of investment in a decentralized value chain are always lower than an integrated one. Thus, the investment level in an integrated value chain is always higher. However, Proposition 1 indicates that this logic does not always hold true in scenarios where network effects can be engineered. This is because to fully utilize the network effects generated by investment requires a relatively large user base. In particular, the manufacturer relies on the retailer to reap the benefit from a larger user base. However, due to double marginalization, the manufacturer has to charge a lower wholesale price $2b - 1$ (i.e., reducing its margin) than the price b it would charge if it were the central planner to incentivize the retailer to penetrate the market. Consequently, a higher b can raise the manufacturer's price (and revenue) more than the central planner's price (and revenue). That is, the marginal profit of investment of the manufacturer is greater than that of the central planner, i.e.,

$$\Pi_M'(b|\hat{\theta}_D^* = 0) = 2 - 2\gamma b > 1 - 2\gamma b = \Pi_I'(b|\hat{\theta}_I^* = 0).$$

Consequently, the decentralized value chain invests more radically in the strength of network effects than the integrated one, a finding that appears novel in the literature.

5 The Generalized Model

The preceding sections highlight that the emergence of novel data network effects can enhance value chain efficiency. To fully leverage these network effects, value chain participants must not only focus on novel strategies like expanding the user network and intensifying the network effect through investment but also on traditional strategies, such as improving cost-efficiency in production—a factor that might be easily overlooked.

In this section, we will delve into a generalized model that incorporates both approaches concurrently. Our goal is to explore the interplay between these methods and to attain a comprehensive understanding of how the value chain can most efficiently monetize network effects.

We begin once more with the centralized case. The central planner's problem is analogous to Equation (5), but with the distinction that b now becomes a decision variable, carrying with it an associated cost of γb^2. Lemma 6 outlines the central planner's optimal strategy.

Lemma 6. *In the generalized model, the central planner's optimal strategy is as follows:*

i. When $c < c_\gamma^I$, $b_I^* = b_I^h := \frac{1}{4\gamma-1}$. Consequently, $\alpha_I^* = \frac{2\gamma-1}{4\gamma-1}$, $\hat{\theta}_I^* = \frac{2\gamma-1}{4\gamma-1}$, and $\Pi_I^*(b_I^h) = \frac{\gamma}{4\gamma-1} - c$.

ii. When $c \geq c_\gamma^I$, $b_I^* = b_I^l \in \left(0, \frac{b_I^h}{2}\right)$, which is the minimal root of

$$b(1-b)^2 = \frac{(1-c)^2}{8\gamma},$$

with ${b_I^l}'(c) < 0$ and ${b_I^l}''(c) > 0$. Therefore, $\alpha_I^* = 0$, $\hat{\theta}_I^* = \frac{1-2b_I^l+c}{2(1-b_I^l)}$, and $\Pi_I^*(b_I^l) = \frac{(1-c)^2}{4(1-b_I^l)} - \gamma {b_I^l}^2$.

The cutoff $c_\gamma^I > 0$ is the unique root of $\Pi_I^*(b_I^h) = \Pi_I^*(b_I^l)$ and decreases with γ.

The findings in Lemma 6 align with those in Lemma 4, further bolstering the robustness of our earlier conclusions. Additionally, note that seeding is optimal (i.e., $\alpha_I^* > 0$) only when c is small (case i), accompanied by the maximum investment in engineering network effects (i.e., $b_I^* = b_I^h$). This means that the decrease in marginal cost motivates the central planner to employ both strategies in conjunction. Moreover, when both strategies are used, the market is fully covered (i.e., $\alpha_I^* = \hat{\theta}_I^*$), which never happens if c is large (case ii). In other words, the improvement of production efficiency (i.e., a smaller c) leads to drastic transformation of the central planner's strategy – shifting from selling to a portion of the market with a relatively low b (case ii), to investing heavily in b and using seeds to fully exploit user base expansion (case i).

Another key observation from comparing Lemmas 4 and 6 is that the strategies for expanding the network and engineering network effects are complementary. Specifically, when seeding becomes viable (as per Lemma 6), the constraint on cost-efficiency for making radical investments (i.e., $\gamma < \tilde{\gamma}^D$) disappear. A similar inference can be drawn for the decentralized scenario when juxtaposing Lemmas 5 and 7.

Lastly, we examine the decentralized case of the generalized model. Lemma 7 below characterizes the equilibrium results.

Lemma 7. *In the generalized model, the equilibrium of the decentralized case is as follows:*

i. When $\gamma < \hat{\gamma}$ and $c < c_\gamma^R$, $b_D^* = b_D^h := \bar{b}$. Consequently, $\alpha_M^* = 0$, $w^* = c^I(b_D^h)$, $\alpha_R^* = \frac{1-b_D^h}{2}$, $\hat{\theta}_D^* = \frac{1-b_D^h}{2}$, $\Pi_M^*(b_D^h) = c^I(b_D^h) - c - \gamma {b_D^h}^2$, and $\Pi_R^*(b_D^h) = \frac{(1+b_D^h)^2}{4} - c^I(b_D^h)$.

ii. When $\gamma \geq \hat{\gamma}$ and $c < c_\gamma^M$, $b_D^* = b_D^m \in (0, b_D^h)$, which is the minimal root of

$$8\gamma b^3 - (32\gamma - 1)b^2 + (32\gamma - 4)b - 5 = 0,$$

with $b_D^{m\prime}(\gamma) < 0$ and $b_D^{m\prime\prime}(\gamma) < 0$. Therefore, $\alpha_M^* = \frac{3(1-b_D^m)}{2(2-b_D^m)}$, $w^* = \frac{1+b_D^m}{2}$, $\alpha_R^* = 0$,

$\hat{\theta}_D^* = \frac{3(1-b_D^m)}{2(2-b_D^m)}$, $\Pi_M^*(b_D^m) = \frac{(1+b_D^m)^2}{4(2-b_D^m)} - c - \gamma {b_D^m}^2$, *and* $\Pi_R^*(b_D^m) = \frac{(1-b_D^m)(1+b_D^m)^2}{4(2-b_D^m)^2}$.

iii. When $c \geq \max\{c_\gamma^R, c_\gamma^M\}$, $b_D^* = b_D^l \in (0, b_D^m)$, *which is the minimal root of*

$$b(1-b)^2 = \frac{(1-c)^2}{16\gamma},$$

with ${b_D^l}'(c) < 0$ *and* ${b_D^l}''(c) > 0$. *Therefore,* $\alpha_M^* = 0$, $w^* = \frac{1+c}{2}$, $\alpha_R^* = 0$, $\hat{\theta}_D^* = \frac{3+c-4b_D^l}{4(1-b_D^l)}$, $\Pi_M^*(b_D^l) = \frac{(1-c)^2}{8(1-b_D^l)} - \gamma {b_D^l}^2$, *and* $\Pi_R^*(b_l^D) = \frac{(1-c)^2}{16(1-b_l^D)}$.

The value of $\hat{\gamma}$ *is approximately* 0.83 *and both* c_γ^R *and* $c_\gamma^M > 0$. *When* $\gamma < \hat{\gamma}$, c_γ^R *is the root of* $\Pi_M^*(b_D^h) = \Pi_M^*(b_D^l)$ *and decreases with* γ*; when* $\gamma \geq \hat{\gamma}$, c_γ^M *is the root of* $\Pi_M^*(b_D^m) = \Pi_M^*(b_D^l)$ *and also decreases with* γ.

Similar to our analysis above, the findings of Lemma 7 have a structure analogous to Lemma 5. However, when considering the seeding strategy and investment of engineering network effects simultaneously, the decentralized case becomes more complex, with up to three potential scenarios: Similar to Lemma 6-i, we show that the seeding strategy only arises when the marginal production cost is sufficiently low (scenario i); However, it remains unknown whether the seeding strategy should be executed by the manufacturer or the retailer.[12] Lemma 7 suggests that it depends on the marginal efficiency of innovation investment, γ. For a small γ, the manufacturer will focus on investing in network externalities and motivates the retailer to expand the market using seeding strategies (i.e., the red curve falls in the region of R-seeding when c is small in Figure 3a); Conversely, if γ is large implying that the network effects are not substantially strong, the retailer has insufficient incentive to employ the seeding strategy. Consequently, the manufacturer has to seed the market and invest in engineering network effects simultaneously (i.e., the red curve falls in the region of M-seeding when c is small in Figure 3b). Based on the discussions above, we

[12]We proved in Lemma 4 that the seeding strategy, when necessary, is carried out by either the manufacturer or the retailer, but not both.

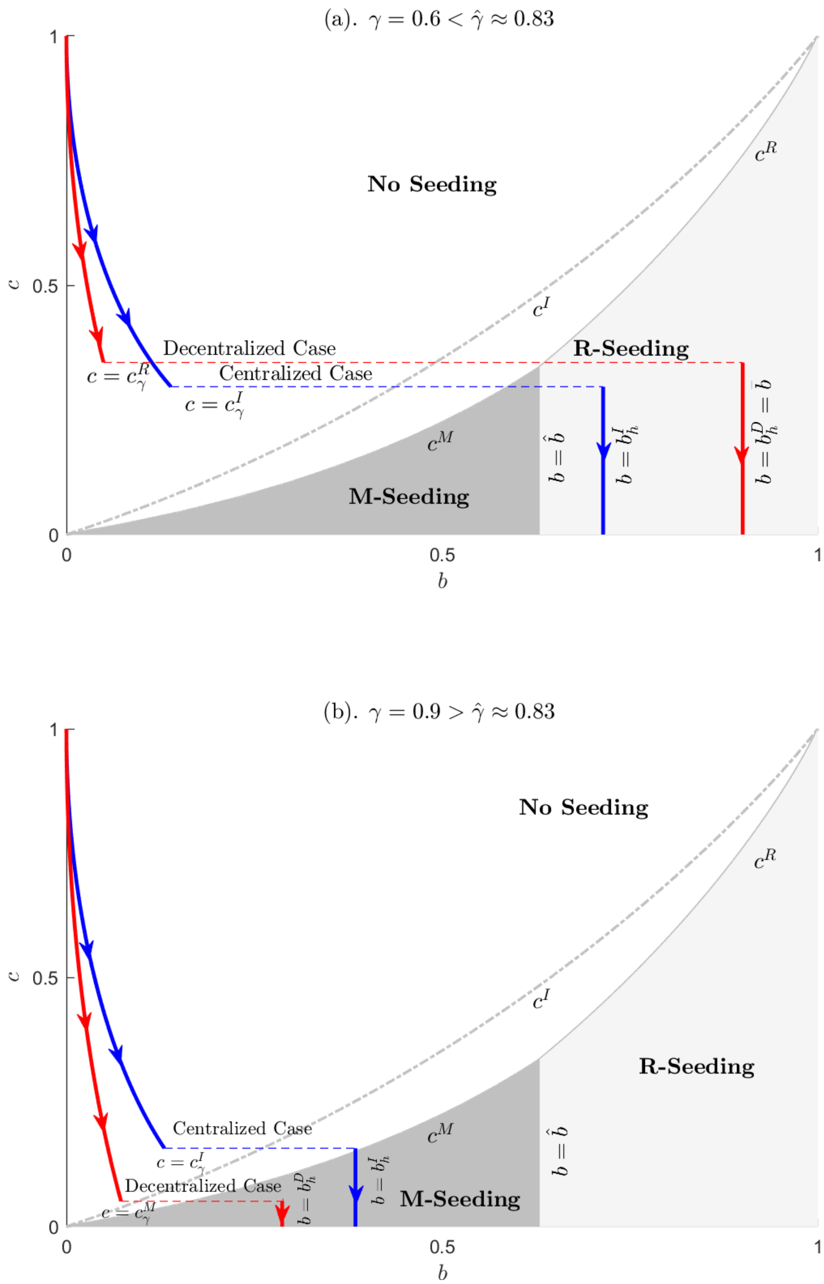

Figure 3: The Optimal Strategy in the Decentralized Case

derive insights on the relationship between the two approaches and present with Proposition 2 below.

Proposition 2. *In the generalized model, both the optimal strength of network effects and the seeding level decrease in the marginal production cost c. Thus, the two approaches com-*

plement each other as production efficiency changes. Formally we have

$$\frac{\partial b_I^*}{\partial c} \leq 0,\ \frac{\partial b_D^*}{\partial c} \leq 0,\ \frac{\partial \alpha_I^*}{\partial c} \leq 0,\ \frac{\partial \alpha_M^*}{\partial c} \leq 0,\ \frac{\partial \alpha_R^*}{\partial c} \leq 0.$$

Proposition 2 is illustrated by Figure 3, where the curves of optimal b gravitate towards the bottom right corners (as the arrows indicate) as c decreases. As the optimal b increases horizontally towards the right side, it enters regions where seeding is optimal. This showcases the complementarity between the two approaches. In summary, Proposition 2 provides a guideline for value chain participants aiming to optimize both approaches concurrently: in cases where the cost of seeding is prohibitively high (i.e., a large c), investing heavily in engineering network effects becomes unprofitable. To fully harness the potential of network effects, there's a need for significant efficiency improvement in production.

Finally, Proposition 3 suggests that Proposition 1 remains robust in the generalized model. This can be demonstrated using a similar marginal benefit analysis approach.[13]

Proposition 3. *In the decentralized case, the strength of the networks effects exceeds that of the centralized case if and only if the equilibrium results in R-seeding. Specifically, when $\gamma < \hat{\gamma}$ and $c < c_\gamma^R$, we have $b_D^* > b_I^*$. For any other admissible pair (γ, c), it follows that $b_D^* < b_I^*$.*

6 Implications for Industry Profit and Social Welfare

In the baseline model, we demonstrated the value of network effects in reducing efficiency loss and improving social welfare (see Corollary 1). Subsequently, in Sections 4 and 5, we

[13] Details of the proof are available in the Appendix.

showed that value chain profits can be further enhanced through strategies to leverage network effects (i.e., seeding and/or engineering network effects). However, it remains to be seen whether these strategies lead to positive impacts on the value chain's performance (i.e., industry profit) and social welfare. We address this question in this section.

To begin, we build upon the baseline model by introducing seeding, while keeping b fixed (as in Section 4.1). Proposition 4 elucidates the effect of seeding on efficiency loss in the decentralized value chain.

Proposition 4. *In contrast to the baseline model, introducing seeding results in non-monotonic impacts on value chain efficiency. Specifically:*

i. When $b \leq \hat{b}$ and $c \in [c^M, c^I)$, or $b > \hat{b}$ and $c \in [c^R, c^I)$, the efficiency loss increases;

ii. When $b \leq \hat{b}$ and $c < c^M$, the efficiency loss decreases;

iii. When $b > \hat{b}$ and $c < c^R$, the efficiency loss is eliminated;

iv. For any other pair (b, c), the seeding strategy has no impact on efficiency loss.

On the one hand, scenario (iii) of Proposition 4 suggests that the previous finding that the efficiency loss can be eliminated — still holds (i.e., $L = 0$ in Corollary 1-i when $b > \frac{3+c}{4}$). In fact, the condition for this equilibrium outcome becomes much less restricted, as the threshold of b drops from $\frac{3+c}{4}$ to $\hat{b} \approx 0.63$, which implies that seeding is beneficial for improving value chain efficiency. This scenario corresponds to the right-bottom corner of "R-seeding" in Figure 2.

Does introducing seeding always generate positive impacts? No. As scenario (i) of Proposition 4 reveals, for all values of b, it is possible (depending on c) that double marginalization will lead to a reduced willingness to use the seeding strategy in the decentralized value chain (compared with the centralized case). This further exacerbates the efficiency loss. This is illustrated in the belt-shaped area between the two curves in Figure 2. In this area, the central planner adopts seeding because $c > c^I$ while seeding is not an equilibrium outcome in the decentralized value chain. To summarize, in the presence of network effects, double marginalization can result in an "under-seeding" distortion and cause greater efficiency loss. We depict these changes in efficiency loss with Figure 4 below. The increasing part of the blue curve (i.e., $b \in (c^{I^{-1}}, c^{M^{-1}})$) represents the scenario where the efficiency loss worsens (i.e., an increasing L).

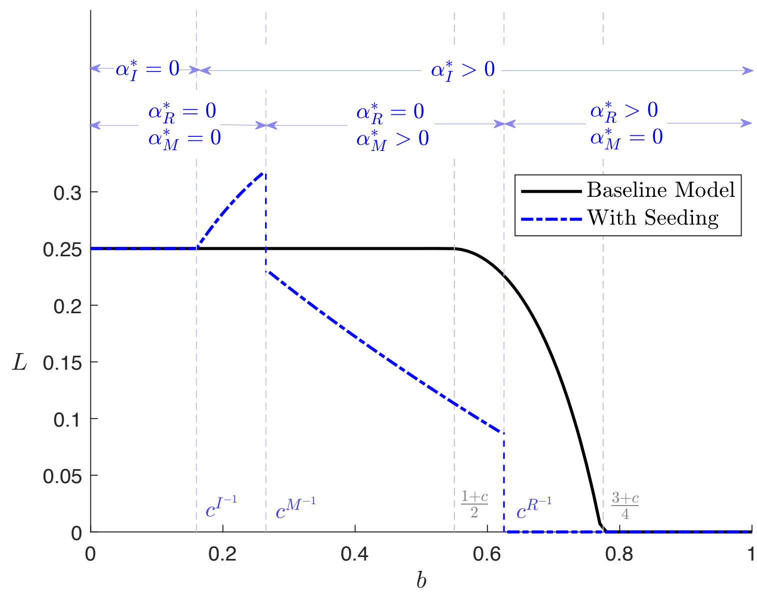

Figure 4: Profits and Efficiency Loss in the Value Chain

Next, we conduct a similar analysis for cases where value chain participants invest in engineering network effects without seeding, as outlined in Section 4.2. Proposition 5 presents the results.

Proposition 5. *The efficiency loss is non-monotonic with respect to γ. Specifically, the efficiency loss increases with γ when $\gamma < \tilde{\gamma}^D$ and $c < \min\{c_\gamma^I, c_\gamma^D\}$, and decreases with γ when $\gamma \geq \tilde{\gamma}^I$.*

The non-monotonicity highlighted in Proposition 5 is tied to the gap between investment levels of centralized and decentralized cases. Specifically, when innovation is relatively efficient (i.e., $\gamma < \tilde{\gamma}^D$), both the centralized and decentralized cases will invest drastically in b such that in equilibrium the market is fully covered. However, as we have shown in Proposition 1, the manufacturer consistently opts for $\bar{b}$, while the central planner will choose $\frac{1}{2\gamma}$, causing an over-investment (i.e., $\bar{b} - \frac{1}{2\gamma}$) which increases in γ (see the left-hand side of Figure 5a). In contrast, if the innovation is less efficient (i.e., $\gamma \geq \tilde{\gamma}^D$), the extent of over-investment (i.e., $b_I^l - b_D^l$) in the decentralized case is decreasing in γ, as shown by Lemmas 4 and 5 and is visualized on the right-hand side of Figure 5a. Therefore, the comprehensive effect on value chain efficiency loss is portrayed in Figure 5b.

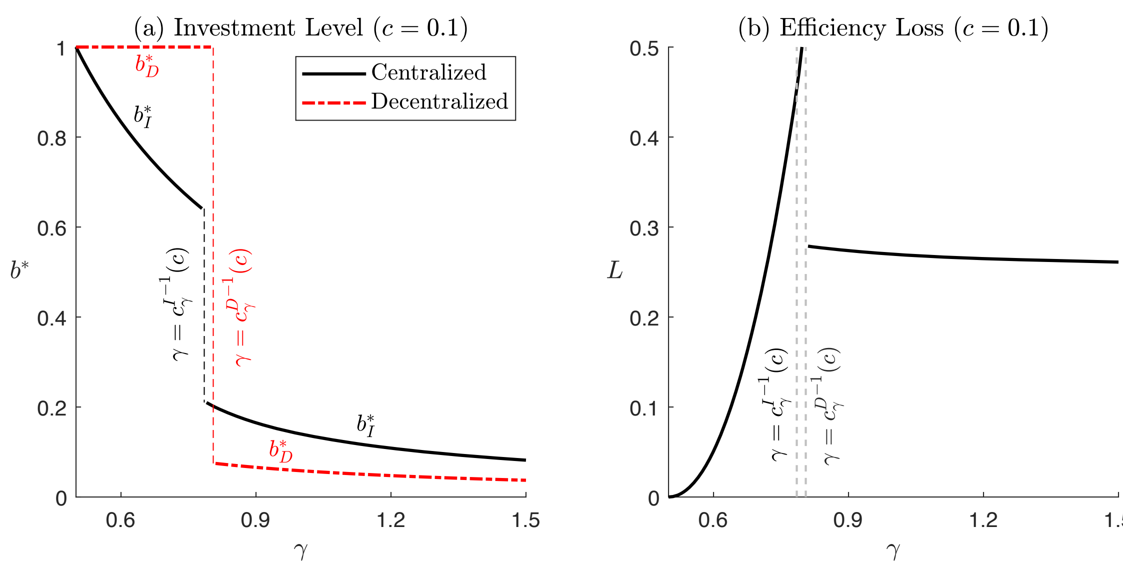

Figure 5: Investment Level, Profits and Efficiency Loss in the Value Chain

Our final analysis is on the social welfare impacts when the value chain exploits the network effects with two approaches. We first establish the benchmark of social optimum. A social welfare maximizer (or the "social planner") chooses the cutoff $\underline{\theta}$ and the investment level b to maximize social welfare as defined below:

$$
\begin{aligned}
W &:= \int_{\underline{\theta}}^{1} [\theta + b(1-\underline{\theta})]d\theta - c(1-\underline{\theta}) - \gamma b^2 \\
&= \left(b - \frac{1}{2}\right)\underline{\theta}^2 - (2b-c)\underline{\theta} + b - \gamma b^2 - c + \frac{1}{2}.
\end{aligned}
$$

The next proposition characterizes the socially optimal allocation $(b_S^*, \underline{\theta}_S^*)$.

Lemma 8. *The socially optimal allocation is as follows:*

(i) When $c < c_\gamma^S$, $b_S^* = b_S^h := \frac{1}{2\gamma}$ *and* $\underline{\theta}_S^* = 0$;

(ii) When $c \geq c_\gamma^S$, $b_S^* = b_S^l \in (0, b_S^h]$, *which is the minimal root of*

$$
b(1-2b)^2 = \frac{(1-c)^2}{2\gamma},
$$

with ${b_S^l}'(c) < 0$ *and* ${b_S^l}''(c) > 0$. *Furthermore,* $\underline{\theta}_S^* = \frac{c-2b_S^l}{1-2b_S^l}$.

The cutoff $c_\gamma^S \in (0, \frac{1}{\gamma}]$ *is the unique root of* $W(b_S^h) = W(b_S^l)$ *and decreases with* γ. *Specifically, when* $\gamma \geq 3$, $c_\gamma^S = \frac{1}{\gamma}$, *and* b_S^* *is continuous in* c *on* $(0,1)$.

Corollary 3. *In the generalized model, the centralized case demonstrates under-investment. Specifically, for all* $\gamma > \frac{1}{2}$ *and* $c \in (0,1)$, *we have* $b_I^* < b_S^*$. *Furthermore,* $c_\gamma^I < c_\gamma^S$.

Thus, we can compare the optimal degree of network effects obtained from our previous analysis with the social optimum benchmark above. Proposition 6 provides a counter-intuitive finding: if the manufacturer is sufficiently efficient in both innovation and production, then the decentralized case can yield higher social welfare than the centralized case.

Proposition 6. *In the generalized model, the decentralized case yields a higher social welfare than the centralized case if* $\frac{1}{2} < \gamma < \frac{3+\bar{b}+\sqrt{\bar{b}^2-10\bar{b}+9}}{8\bar{b}}$ *and* $c < \min\{c_\gamma^I, c_\gamma^R\}$.

The key takeaway from Proposition 6 is that the advent of network effects can significantly alter the conventional understanding that centralized value chains yield higher social welfare. In the absence of network effects, decentralized value chains tend to overprice, which reduces market demand and results in lower social welfare. However, with network effects, attempts to monetize them can lead to additional investment in engineering network effects. As more customers become adopters (either paying or seeded) and derive added value from network benefits, decentralized value chains may achieve even greater social welfare. It's noteworthy that Corollary 1 has shown that, with a fixed b, decentralized value chains can at best match the social welfare of the centralized case. Thus, Proposition 6 takes this a step further, suggesting that decentralized value chains can surpass centralized ones in social welfare when various strategies are employed to harness network effects.

7 Conclusion

As connected products increasingly attract attention from researchers, we are among the first to investigate their value chain. We find that the network effects among these products have profound impacts. Our analysis provides insights from four angles.

First, as physical products incorporate more connectivity features, the network effects they generate offer new opportunities for profit. Using the benchmark model (Section 3), we demonstrate that value chain participants benefit from these network effects. In Section 4, we explore two main strategies: 1) seeding to expand the network and 2) investing to fortify the connection. We present closed-form solutions and optimal conditions for both strategies, validating their effectiveness with the generalized model (Section 5). Notably, we find that employing seeding and investment concurrently can complement each other; a substantial investment in engineering network effects is more likely to be optimal if targeted seeding is attainable. These findings present a comprehensive strategy for value chain participants to exploit the rising prominence of network effects.

Second, at the industry level, there is encouraging news: strategic coordination is achievable. Traditional products often necessitate intricate contract designs among value chain members to counteract price distortions and efficiency losses. Contrarily, our results reveal that inefficiencies for connected products can be entirely resolved, even under the wholesale price contract. Furthermore, when network effects are endogenous, the ideal level might be more pronounced in a decentralized value chain, suggesting a deeper dive into connectivity features. This stems from manufacturers' inclination for more potent network effects (compared to the central planner's preference), allowing for a higher wholesale price. Clearly, network effects introduce more intricate dynamics within the value chain.

Third, our results underscore the societal benefits of revolutionizing the network effect. Especially as marginal costs decrease, seeding strategies to amplify the network appear optimal, driving rapid connectivity innovation. This mirrors observed innovations in fields like robotic technologies, autonomous driving, and voice assistants. Our model sheds light on these bold, occasionally unconventional, innovation efforts.

Lastly, the social planning approach is evolving, as indicated in our welfare analysis. The emerging paradigm suggests that the triumph of connected products hinges on an initial expansive user network, ultimately favoring the broader market. Crucially, as illustrated in our Proposition 6, welfare in a decentralized scenario can surpass that of a centralized one.

In summation, our paper paves the way for a promising research trajectory. It will be enriching to examine data collection and algorithm training from a value chain angle, especially when the connected product integrates artificial intelligence modules (Gurkan and de Véricourt 2022). Other valuable avenues include user privacy issues (Sun and Ji 2022) and the nuances of network topology (Dou et al. 2013) .

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Appendix

A Notation

b	Strength of network effect, $b \in (0,1)$
c	Marginal production cost, $c \in (0,1)$
w	Wholesale price set by the manufacturer
p	Market price set by the retailer or the central planner
$\hat{\theta}$	Marginal customer type
α_I	Seeding level chosen by the central planner
α_M (α_R)	Seeding level chosen by the manufacturer (retailer), $\alpha_M, \alpha_R \geq 0$ and $\alpha_M + \alpha_R \leq 1$
Π_I	Profit of the central planner
Π_M (Π_R)	Profit of the manufacturer (retailer)
c^I	Seeding threshold for the central planner
c^M (c^R)	Seeding threshold for the manufacturer (the retailer)
	For endogenous network effects (Sections 4.2, 5 and 6)
γ	Coefficient of marginal innovation cost for engineering network effect, $\gamma > \frac{1}{2}$
b_I	Strength of network effect chosen by the central planner
b_D	Strength of network effect chosen by the manufacturer
$\tilde{\gamma}^I$	Investment threshold for the central planner without seeding
$\tilde{\gamma}^D$	Investment threshold for the manufacturer without seeding
$\hat{\gamma}$	Investment threshold for the manufacturer in the generalized model
c^I_γ	Full coverage threshold for the central planner given γ with or without seeding
c^D_γ	Full coverage threshold for the manufacturer given γ without seeding
$c^M_\gamma (c^R_\gamma)$	Seeding threshold for the manufacturer (the retailer) given γ
L	The efficiency loss ratio
W	Social welfare, with W_I (W_D) being that of the centralized (decentralized) case
b_S	Socially optimal strength of network effect
$\underline{\theta}_S$	Socially optimal marginal customer type

B Proofs

Proof of Lemma 1

Proof. We first consider the centralized case. Note that Π_I is strictly concave in $\hat{\theta}_I$. Thus, there are two cases. If $\hat{\theta}_I^* > 0$, then the first-order condition implies that $\hat{\theta}_I^* = \frac{1+c-2b}{2(1-b)}$, which is positive if $b < \frac{1+c}{2}$; otherwise, $\hat{\theta}_I^* = 0$. The remainder of statement (*i*) is thus immediate. Next, we consider the decentralized case. Note that given w, the retailer faces the same problem as the central planner except that the marginal cost equals w. Thus, analogously, the retailer chooses $\hat{\theta}_D = \frac{1+w-2b}{2(1-b)}$ if $b < \frac{1+w}{2}$, and $\hat{\theta}_D = 0$ otherwise. It follows that the manufacturer's profit is given by the following function of w.

$$\Pi_M(w) = \begin{cases} \frac{(w-c)(1-w)}{2(1-b)} & \text{if } w > 2b-1; \\ w - c & \text{if } w \leq 2b-1. \end{cases}$$

Clearly, if $w^* \leq 2b-1$, then $w^* = 2b-1$ and thus $\Pi_M = 2b-1-c$. If $w^* \neq 2b-1$, then $w^* = \frac{1+c}{2} > 2b-1$ always holds, meaning that $b < \frac{3+c}{4}$. It follows that $w^* = 2b-1$ if $b \geq \frac{3+c}{4}$ and $w^* = \frac{1+c}{2}$ otherwise. Then, the remainder of the lemma is immediate. □

Proof of Lemma 2

Proof. Note that at the optimum $1 > \hat{\theta}_I^* \geq \alpha_I^* \geq 0$. Thus, the Lagrangian is given by

$$\mathcal{L} = [(1-b)\hat{\theta}_I + (1+\alpha_I)b - c](1-\hat{\theta}_I) - c\alpha_I + \lambda(\hat{\theta}_I - \alpha_I) + \mu\alpha_I,$$

where λ and μ are the Lagrangian multipliers. Equation (5) implies that the determinant of the Hessian matrix of Π_I is $-b^2 < 0$; thus, $(\hat{\theta}_I, \alpha_I)$ cannot be an interior solution. Then, we need to consider three cases: (*i*) $\hat{\theta}_I^* = \alpha_I^* = 0$, (*ii*) $\hat{\theta}_I^* = \alpha_I^* > 0$, and (*iii*) $\hat{\theta}_I^* > \alpha_I^* = 0$. The first-order conditions of α_I and $\hat{\theta}_I$ are given by, respectively,

$$\frac{\partial \mathcal{L}}{\partial \alpha_I} = (1-\hat{\theta}_I)b - c - \lambda + \mu = 0, \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\theta}_I} = -2(1-b)\hat{\theta}_I - (2+\alpha_I)b + 1 + c + \lambda = 0. \tag{A.2}$$

First, if $\hat{\theta}_I^* = \alpha_I^* = 0$, then $\lambda, \mu > 0$. Adding (A.1) and (A.2) up, we have $\mu = b - 1 < 0$, a contradiction. Second, if $\hat{\theta}_I^* = \alpha_I^* > 0$, then $\lambda > 0$ and $\mu = 0$. Adding (A.1) and (A.2) up, we have $\hat{\theta}_I^* = \alpha_I^* = \frac{1-b}{2}$. It follows that $p_I^* = \frac{1+b}{2}$ and $\Pi_I^* = \frac{(1+b)^2}{4} - c$. Third, if $\hat{\theta}_I^* > \alpha_I^* = 0$, then $\lambda = 0$ and $\mu > 0$. Solving (A.2), we have $\hat{\theta}_I^* = \frac{1+c-2b}{2(1-b)}$. Substituting $\hat{\theta}_I^*$ into (A.1), we have $\mu = \frac{(2-b)c-b}{2(1-b)}$. Thus, $\mu > 0$ if $c > \frac{b}{2-b}$; if $c \leq \frac{b}{2-b}$, then it must be that $\hat{\theta}_I^* = \alpha_I^* = \frac{1-b}{2}$.

When $\mu > 0$ holds, we have $p_I^* = \frac{1+c}{2}$ and $\Pi_I^* = \frac{(1-c)^2}{4(1-b)}$. It follows that when $c > \frac{b}{2-b}$,

$$\frac{(1-c)^2}{4(1-b)} \geq \frac{(1+b)^2}{4} - c \Longleftrightarrow c \geq c^I := 1 - 2(1-b) + (1-b)^{\frac{3}{2}} \in (0,1).$$

It can be shown that $c^I > \frac{b}{2-b}$ for $b \in (0,1)$. Thus, if $c \geq c^I$, then $\hat{\theta}_I^* = \frac{1+c-2b}{2(1-b)}$ and $\alpha_I^* = 0$; otherwise, $\hat{\theta}_I^* = \alpha_I^* = \frac{1-b}{2}$. The remainder of the lemma then follows immediately. $\square$

Proof of Lemma 3

Proof. As a first step, we prove two useful lemmas.

Lemma A1. *Suppose that in equilibrium $\alpha_M^* = 0$, then*

i. When $c < c^R$, $w^* = c^I$, $\alpha_R^* = \frac{1-b}{2}$ and $\hat{\theta}_D^* = \frac{1-b}{2}$;

ii. When $c \geq c^R$, $w^* = \frac{1+c}{2}$, $\alpha_R^* = 0$ and $\hat{\theta}_D^* = \frac{3+c-4b}{4(1-b)}$.

Proof. Since $\alpha_M = 0$, the retailer faces the same problem as the central planner in Lemma 2 except that the marginal cost equals w. Thus, given some w, the retailer's best response satisfies that if $w \leq c^I$, $\alpha_R(w) = \frac{1-b}{2}$ and $\hat{\theta}(w) = \frac{1-b}{2}$; otherwise, $\alpha_R(w) = 0$ and $\hat{\theta}(w) = \frac{1+w-2b}{2(1-b)}$. It follows that the manufacturer's profit is given by

$$\Pi_M(w|\alpha_M = 0) = (w-c)[\alpha_R(w) + 1 - \hat{\theta}(w)] = \begin{cases} \frac{(1-w)(w-c)}{2(1-b)} & \text{if } w > c^I; \\ w - c & \text{if } w \leq c^I. \end{cases}$$

Clearly, if $w^* \leq c^I$, then $w^* = c^I$ and thus $\Pi_M = c^I - c$; otherwise, we have to check whether the unconstrained optimizer, $\frac{1+c}{2}$, is attainable. Note that $\frac{1+c}{2} > c^I$ if and only if $c > \hat{c} := 1 - 4(1-b) + 2(1-b)^{\frac{3}{2}}$. It is easy to show that $\hat{c} < c^I$. Hence, if $w > c^I$, then $w^* = \frac{1+c}{2}$ and $\Pi_M = \frac{(1-c)^2}{8(1-b)}$. Thus, when $c > \hat{c}$,

$$\frac{(1-c)^2}{8(1-b)} \geq c^I - c \Longleftrightarrow c \geq c^R := \max\{0, 1 - 4(1-b) + 2\sqrt{2}(1-b)^{\frac{5}{4}}\}.$$

It can be easily verified that $c^R < c^I$. In summary, when $c < c^R$, $w^* = c^I$, and thus, $\alpha_R^* = \frac{1-b}{2}$ and $\hat{\theta}_D^* = \frac{1-b}{2}$; when $c \geq c^R$, $w^* = \frac{1+c}{2}$, and thus, $\alpha_R^* = 0$ and $\hat{\theta}_D^* = \frac{3+c-4b}{4(1-b)}$. $\square$

Lemma A2. *Suppose that in equilibrium $\alpha_M^* > 0$, then it must be that $c < c^M < c^I$, and thus, $\alpha_M^* = \hat{\theta}_D^* = \frac{3(1-b)}{2(2-b)}$, $w^* = \frac{1+b}{2}$ and $\alpha_R^* = 0$.*

Proof. Given some α_M and w, the retailer chooses α_R and $\hat{\theta}_D$ to maximize Equation (6) subject to that $\hat{\theta}_D \in [\alpha_M + \alpha_R, 1]$. The Lagrangian is given by

$$\mathcal{L} = [(1-b)\hat{\theta}_D + (1+\alpha_M+\alpha_R)b - w](1-\hat{\theta}_D) - w\alpha_R + \lambda(\hat{\theta}_D - \alpha_M - \alpha_R) + \mu\alpha_R.$$

The first-order conditions of α_R and $\hat{\theta}_D$ are given by, respectively,

$$\frac{\partial \mathcal{L}}{\partial \alpha_R} = (1-\hat{\theta}_D)b - w - \lambda + \mu = 0, \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\theta}_D} = -2(1-b)\hat{\theta}_D - (2+\alpha_M+\alpha_R)b + 1 + w + \lambda = 0. \tag{A.4}$$

We consider two cases. First, if $\alpha_R > 0$, then $\mu = 0$. Moreover, note that the determinant of the Hessian matrix of Π_R is $-b^2 < 0$; thus, $(\hat{\theta}_D, \alpha_R)$ cannot be an interior solution. This means that $\hat{\theta}_D = \alpha_M + \alpha_R$. Substituting and adding (A.3) and (A.4) up, we have $\hat{\theta}_D = \frac{1-b}{2}$, which is independent of w and α_M. Then, substituting $\hat{\theta}_D$ and α_R into (7), we have $\Pi_M = (w-c)(1-\alpha_M) - c\alpha_M$. Clearly, the optimal α_M is zero. This implies that if in equilibrium $\alpha_M^* > 0$, then we must have $\alpha_R^* = 0$. Since $\alpha_R^* = 0$, if $\hat{\theta} > \alpha_M$, then by (A.4), we have $\hat{\theta} = \frac{1+w-(2+\alpha_M)b}{2(1-b)}$, which is larger than α_M if $w > (2-b)\alpha_M + 2b - 1$, meaning that if $w \leq (2-b)\alpha_M + 2b - 1$, then $\hat{\theta}_D = \alpha_M$. Thus, the manufacturer's profit is given by

$$\Pi_M(w, \alpha_M) = (w-c)(1-\hat{\theta}_D) - c\alpha_M = \begin{cases} \frac{(w-c)(1-w+b\alpha_M)}{2(1-b)} - c\alpha_M & \text{if } w > (2-b)\alpha_M + 2b - 1; \\ w(1-\alpha_M) - c & \text{if } w \leq (2-b)\alpha_M + 2b - 1. \end{cases}$$

Clearly, if $w^* \leq (2-b)\alpha_M^* + 2b - 1$, then $w^* = (2-b)\alpha_M^* + 2b - 1$. Substituting w^* into Π_M and by the first-order condition of α_M, we have $\alpha_M^* = \frac{3(1-b)}{2(2-b)}$, and thus, $w^* = \frac{1+b}{2}$ and $\Pi_M^* = \frac{(1+b)^2}{4(2-b)} - c$. If $w > (2-b)\alpha_M + 2b - 1$, then $\Pi_M = \frac{(w-c)(1-w+b\alpha_M)}{2(1-b)} - c\alpha_M$; thus, the determinant of the Hessian matrix of Π_M is $-\frac{b^2}{4(1-b)^2} < 0$. This means that (w, α_M) cannot be an interior solution. Since $w > (2-b)\alpha_M + 2b - 1$, $\alpha_M < \hat{\theta}_D$. Clearly, w^* cannot be a boundary solution, thus it must be $\alpha_M^* = 0$, a contradiction. Therefore, if $\alpha_M^* > 0$, then we must have $\alpha_M^* = \hat{\theta}_D^* = \frac{3(1-b)}{2(2-b)}$, $w^* = \frac{1+b}{2}$ and $\alpha_R^* = 0$. It follows that $\Pi_M^* = \frac{(1+b)^2}{4(2-b)} - c$.

It remains to show that if $\alpha_M^* > 0$, then $c < c^M$. From the proof of Lemma A1, we have that if the manufacturer chooses $\alpha_M = 0$ and $w = \frac{1+c}{2} > c^I$, then $\Pi_M = \frac{(1-c)^2}{8(1-b)}$. Thus, the strategy $(\alpha_M = 0, w = \frac{1+c}{2})$ is more profitable than $(\alpha_M = \frac{3(1-b)}{2(2-b)}, w = \frac{1+b}{2})$ if

$$\frac{(1-c)^2}{8(1-b)} \geq \frac{(1+b)^2}{4(2-b)} - c \iff c \geq c^M := 1 - 4(1-b) + 3(1-b)\sqrt{\frac{2-2b}{2-b}}.$$

It can be easily verified that $c^M < c^I$. Thus, if in equilibrium $\alpha_M^* > 0$, then $c < c^M$. $\square$

Lemmas A1 and A2 imply that when $c \geq \max\{c^R, c^M\}$, the equilibrium outcome is given by $\alpha_M^* = 0$, $w^* = \frac{1+c}{2}$, $\alpha_R^* = 0$ and $\hat{\theta}_D^* = \frac{3+c-4b}{4(1-b)}$. In contrast, when $c < \min\{c^R, c^M\}$, the equilibrium outcome has two candidates:

i. $\alpha_M^* = 0$, $w^* = c^I$, $\alpha_R^* = \frac{1-b}{2}$ and $\hat{\theta}^* = \frac{1-b}{2}$;

ii. $\alpha_M^* = \frac{3(1-b)}{2(2-b)}$, $w^* = \frac{1+b}{2}$, $\alpha_R^* = 0$ and $\hat{\theta}^* = \frac{3(1-b)}{2(2-b)}$.

Note that $c^R > c^M$ if and only if $b > \hat{b}$. Thus, when $b > \hat{b}$, $c^R > c^M$ and $c^I - c > \frac{(1+b)^2}{4(2-b)} - c$. This implies that if $c \leq c^R$, then the manufacturer's optimal strategy is $(\alpha_M^* = 0, w^* = c^I)$. Consequently, the equilibrium outcome is given by the above (i). In contrast, when $b \leq \hat{b}$, $c^R \leq c^M$ and $c^I - c \leq \frac{(1+b)^2}{4(2-b)} - c$ with strict inequality for $b > \hat{b}$. Thus, if $c \leq c^M$, then the manufacturer's optimal strategy is $(\alpha_M^* = \frac{3(1-b)}{2(2-b)}, w^* = \frac{1+b}{2})$; thus, the equilibrium outcome is given by the above (ii). Then, the remainder of the lemma follows immediately. $\square$

Proof of Lemma 4

Proof. We consider two cases. First, if $b_I^* > \frac{1+c}{2}$, then $\Pi_I^*(b_I^*) = b - c - \gamma b_I^{*2}$. Since $\Pi_I^*(b_I^*)$ is strictly concave, if $c < \frac{1}{\gamma} - 1$, then $b_I^* = \frac{1}{2\gamma} := b_I^h$; otherwise, $b_I^* \leq \frac{1+c}{2}$. Second, if $b_I^* \leq \frac{1+c}{2}$, then $\Pi_I^*(b_I^*) = \frac{(1-c)^2}{4(1-b_I^*)} - \gamma b_I^{*2}$. Thus, the first-order condition of b is

$$\Pi_I^{*\prime}(b) = \frac{(1-c)^2}{4(1-b)^2} - 2\gamma b = \frac{2\gamma}{(1-b)^2}\left[\frac{(1-c)^2}{8\gamma} - b(1-b)^2\right] = 0. \tag{A.5}$$

Note that $b(1-b)^2$ is single-peaked on $[0, \bar{b}]$ and is (locally) maximized at $b = \frac{1}{3}$, with a maximum equal to $\frac{4}{27}$. Thus, (A.5) has a solution on $[0, \bar{b}]$ if and only if $c \geq 1 - \frac{4}{3}\sqrt{\frac{2\gamma}{3}}$, which is less than $\frac{1}{\gamma} - 1$. If $c \leq 1 - \frac{4}{3}\sqrt{\frac{2\gamma}{3}}$, then the central planner would choose $\bar{b} > \frac{1+c}{2}$; thus, it must be that $b_I^* = b_I^h$. Suppose $c > 1 - \frac{4}{3}\sqrt{\frac{2\gamma}{3}}$. It follows from (A.5) that if $b_I^* < \frac{1+c}{2}$, then it must be the minimal root of (A.5) on $(0, \bar{b})$, denoted b_I^l, and that $b_I^l < \frac{1}{3} < b_I^h$. Then, applying the implicit function theorem to (A.5), we have

$$\frac{db_I^l}{dc} = \frac{(1-c)(1-b_I^l)}{(1-c)^2 - 4\gamma(1-b_I^l)^3} = \frac{1-c}{4\gamma(1-b_I^l)(3b_I^l - 1)} < 0. \tag{A.6}$$

The second equality is due to (A.5) and the inequality is due to that $0 < b_I^l < \frac{1}{3}$; thus, $b_I^{l\prime}(c) < 0$. By (A.6), $b_I^{l\prime\prime}(c)$ has the same sign as $(1-b_I^l)(1-3b_I^l) - (1-c)(4-6b_I^l)b_I^{l\prime}(c)$, which is positive because $b_I^{l\prime}(c) < 0$ and $0 < b_l^I < \frac{1}{3}$; thus, $b_I^{l\prime\prime}(c) > 0$.

Then, we compare $\Pi_I^*(b_I^h)$ with $\Pi_I^*(b_I^l)$. By the envelope theorem, $\frac{d\Pi_I^*(b_I^l)}{dc} = -\frac{(1-c)}{2(1-b_I^l)}$. Since $c \in (0,1)$ and $b_I^l \in (0, \frac{1}{3})$, $\frac{d\Pi_I^*(b_I^l)}{dc} \in (-\frac{3}{4}, 0)$. Moreover, $\frac{d^2\Pi_I^*(b_I^l)}{dc^2} = \frac{(1-b_I^l)-(1-c)b_I^{l\prime}(c)}{2(1-b_I^l)^2} > 0$.

The inequality is because $b_I^{l\,\prime}(c) < 0$. Thus, $\Pi_I^*(b_I^l)$ is decreasing and convex in c. Note that $\frac{d\Pi_I^*(b_I^h)}{dc} = -1$. Thus, $\Pi_I^*(b_I^h)$ and $\Pi_I^*(b_I^l)$ intersect at most once in terms of c. If $c = 0$, then by (A.5), $\Pi_I^*(b_I^l) = \frac{2-3b_I^l}{8(1-b_I^l)^2}$, which is less than $\frac{9}{32}$ since $b_I^l < \frac{1}{3}$. Since $\Pi_I^*(b_I^h) = \frac{1}{4\gamma}$, if $\gamma \leq \frac{8}{9}$, then $\Pi_I^*(b_I^h) > \Pi_I^*(b_I^l)$ at $c = 0$. On the other hand, for c close to 1, $\Pi_I^*(b_I^h) < 0 < \Pi_I^*(b_I^l)$. It follows from continuity that $\Pi_I^*(b_I^h)$ and $\Pi_I^*(b_I^l)$ intersect at some $c_\gamma^I \in (0,1)$ such that for all $c < c_\gamma^I$, $\Pi_I^*(b_h^I) > \Pi_I^*(b_l^I)$ and thus $b_I^* = b_I^h$; otherwise, $b_I^* = b_I^l$. In contrast, for sufficiently large γ, say $\gamma \geq 1$, $\Pi_I^*(b_h^I) < \Pi_I^*(b_l^I)$ for all c and thus $c_\gamma^I = 0$. Moreover, by the envelope theorem, $\frac{d\Pi_I^*(b_I^l)}{d\gamma} = -{b_I^l}^2 > -{b_I^h}^2 = \frac{d\Pi_I^*(b_I^h)}{d\gamma}$. It thus follows from the continuity of Π_I^* that there exists a unique cutoff $\tilde{\gamma}^I$ such that $c_\gamma^I \in (0,1)$ if $\gamma < \tilde{\gamma}^I$ and $c_\gamma^I = 0$ if $\gamma \geq \tilde{\gamma}^I$. In particular, when $c_\gamma^I \in (0,1)$, by the implicit function theorem, we have $\frac{dc_\gamma^I}{d\gamma} = -\frac{{b_I^h}^2 - {b_I^l}^2}{\hat{\theta}_I^*(b_I^l, c_\gamma^I)}$, where $\hat{\theta}_I^*(b_I^l, c_\gamma^I)$ is the cutoff type when $b = b_I^l$ and $c = c_\gamma^I$. Since $b_I^h > b_I^l$, we have $\frac{dc_\gamma^I}{d\gamma} < 0$. Therefore, c_γ^I is nonincreasing in γ. Specifically, c_γ^I is decreasing in γ if $\gamma < \tilde{\gamma}^I$ and is fixed at 0 otherwise. The remainder of the lemma follows immediately. □

Proofs of Lemma 5

Proof. We consider two cases. First, if $b_D^* > \frac{3+c}{4}$, then $\Pi_M^*(b_D^*) = 2b_D^* - 1 - c - \gamma {b_D^*}^2$. Since $\Pi_M^*(b_D^*)$ is strictly concave, if $c < \frac{4}{\gamma} - 3$, then $b_D^* = \min\{\frac{1}{\gamma}, \bar{b}\}$; otherwise, $b_D^* \leq \frac{3+c}{4}$. But if $\frac{1}{\gamma} \leq \bar{b}$, then $\Pi_M^*(b_D^*) = \frac{1}{\gamma} - 1 - c < 0$, a contradiction; thus, $b_D^* = \bar{b} := b_D^h$ only if $\gamma \leq 1$. Second, if $b_D^* \leq \frac{3+c}{4}$, then $\Pi_M^*(b_D^*) = \frac{(1-c)^2}{8(1-b_D^*)} - \gamma {b_D^*}^2$. Thus, the first-order condition of b is

$$\Pi_M^{*\,\prime}(b) = \frac{(1-c)^2}{8(1-b)^2} - 2\gamma b = \frac{2\gamma}{(1-b)^2}\left[\frac{(1-c)^2}{16\gamma} - b(1-b)^2\right] = 0. \tag{A.7}$$

Analogous to the proof of Lemma 4, one can easily show that if $b_D^* \leq \frac{3+c}{4}$, then b_D^* is the minimal root of (A.7), denoted b_D^l, so that $b_D^l < \frac{1}{3}$, $b_D^{l\,\prime}(c) < 0$ and $b_D^{l\,\prime\prime}(c) > 0$. Moreover, $\Pi_M^*(b_D^l)$ is decreasing and convex in c, with $\frac{d\Pi_M^*(b_D^l)}{dc} = -\frac{(1-c)}{4(1-b_D^l)} > -\frac{3}{8}$, while $\frac{d\Pi_M^*(b_D^h)}{dc} = -1$. Thus, $\Pi_M^*(b_D^h)$ and $\Pi_M^*(b_D^l)$ intersect at most once in terms of c. Similarly, we can show that for sufficiently small γ, $\Pi_M^*(b_D^h)$ and $\Pi_M^*(b_D^l)$ intersect at some $c_\gamma^D \in (0,1)$ such that for all $c < c_\gamma^D$, $\Pi_M^*(b_h^D) > \Pi_M^*(b_l^D)$ and thus $b_D^* = b_D^h$; otherwise, $b_D^* = b_D^l$. Moreover, we have $\frac{dc_\gamma^D}{d\gamma} = -\frac{{b_D^h}^2 - {b_D^l}^2}{\hat{\theta}_D^*(b_D^l, c_\gamma^D)} < 0$, where $\hat{\theta}_D^*(b_D^l, c_\gamma^D)$ is the cutoff type when $b = b_D^l$ and $c = c_\gamma^D$. In contrast, for sufficiently large γ, $\Pi_M^*(b_h^D) < \Pi_M^*(b_l^D)$ for all c and thus $c_\gamma^D = 0$. In addition, we have $\frac{d\Pi_M^*(b_D^l)}{d\gamma} = -{b_D^l}^2 > -{b_D^h}^2 = \frac{d\Pi_M^*(b_D^h)}{d\gamma}$. It thus follows from the continuity of Π_M^* that there exists a unique cutoff $\tilde{\gamma}^D$ such that c_γ^D is decreasing in γ if $\gamma < \tilde{\gamma}^D$ and is fixed at 0 if $\gamma \geq \tilde{\gamma}^D$. Lastly, we show that $\tilde{\gamma}^D < \tilde{\gamma}^I$. By the the proof of Lemma 4, we have $\tilde{\gamma}^I \geq \frac{8}{9}$. Suppose $\gamma = \frac{8}{9}$ and $c = 0$. It can be shown that $\Pi_M^*(b_D^h) = 2\bar{b} - 1 - \gamma\bar{b}^2 \approx \frac{1}{9}$ whereas $\Pi_M^*(b_D^l) = \frac{(1-c)^2}{8(1-b_D^l)} - \gamma {b_D^l}^2 \approx 0.13 > \frac{1}{9}$. Since $\Pi_M^*(b_D^h)$

is steeper than $\Pi^*_M(b^l_D)$ with respect to c, we have $\Pi^*_M(b^h_D) < \Pi^*_M(b^l_D)$ for all c when $\gamma = \frac{8}{9}$. This implies that $\tilde{\gamma}^D < \frac{8}{9} \leq \tilde{\gamma}^I$. $\square$

Proof of Proposition 1

Proof. There are three cases. First, if $\gamma < \tilde{\gamma}^D$ and $c < \min\{c^I_\gamma, c^D_\gamma\}$, then $b^*_D = \bar{b} > \frac{1}{2\gamma} = b^*_I$. Second, if $\tilde{\gamma}^D \leq \gamma < \tilde{\gamma}^I$ and $c < c^I_\gamma$, then $b^*_D = b^l_D$ and $b^*_I = \frac{1}{2\gamma}$. By the proof of Lemma 4, we have $b^l_D < \frac{1}{3}$ and $\gamma < 1$. Thus, $b^*_I > b^*_D$. Finally, if $\tilde{\gamma}^D \leq \gamma < \tilde{\gamma}^I$ and $c \geq c^I_\gamma$, or if $\gamma > \tilde{\gamma}^I$, then $b^*_D = b^l_D$ and $b^*_I = b^l_I$. It follows from the definitions of b^l_I and b^l_D that $b^l_I > b^l_D$. $\square$

Proof of Lemma 6

Proof. We consider two cases. First, if $b^*_I > c^{I^{-1}}(c)$, then $\Pi^*_I(b^*_I) = \frac{(1+b)^2}{4} - c - \gamma {b^*_I}^2$. Since $\Pi^*_I(b^*_I)$ is strictly concave and $c^{I^{-1}}(c)$ is increasing, if $c < c^{I^{-1}}(\frac{1}{4\gamma-1})$, then $b^*_I = \frac{1}{4\gamma-1} := b^h_I$ and thus $\Pi^*_I(b^h_I) = \frac{\gamma}{4\gamma-1} - c$; otherwise, b^*_I is given by b^l_I characterized in Lemma 4 and thus $\Pi^*_I(b^l_I) = \frac{(1-c)^2}{4(1-b^l_I)} - \gamma {b^l_I}^2$. By the proof of Lemma 4, we have $\frac{d\Pi^*_I(b^l_I)}{dc} > -\frac{3}{4} > -1 = \frac{d\Pi^*_I(b^h_I)}{dc}$. Thus, $\Pi^*_I(b^h_I)$ and $\Pi^*_I(b^l_I)$ intersect at most once in terms of c. By Lemma 2, we have that the central planner prefers seeding to no seeding for c close to 0 and the other way around for c close to 1. Thus, for any $\gamma > \frac{1}{2}$, $\Pi^*_I(b^h_I)$ and $\Pi^*_I(b^l_I)$ intersect at some $c^I_\gamma \in (0,1)$ such that for all $c < c^I_\gamma$, $\Pi^*_I(b^I_h) > \Pi^*_I(b^I_l)$ and thus $b^*_I = b^h_I$; otherwise, $b^*_I = b^l_I$. Moreover, we claim that $b^h_I > 2b^l_I$. Suppose not, then $\frac{b^h_I}{2} \leq b^l_I < c^{I^{-1}}(c)$, and thus,

$$\Pi^{*\prime}_I(\frac{b^h_I}{2}) = -\frac{1-c}{2(1-\frac{b^h_I}{2})} - 2\gamma\frac{b^h_I}{2} < -\frac{1-c^I(\frac{b^h_I}{2})}{2(1-\frac{b^h_I}{2})} - 2\gamma\frac{b^h_I}{2} = \frac{32\gamma-11}{4(8\gamma-2)} - \sqrt{\frac{8\gamma-3}{8\gamma-2}}. \tag{A.8}$$

The inequality is due to that $c^I(\frac{b^h_I}{2}) > c$ since $\frac{b^h_I}{2} < c^{I^{-1}}(c)$. The equality follows from substituting $\frac{b^h_I}{2}$. It can be verified that the RHS of (A.8) is negative for all $\gamma > \frac{1}{2}$. This means that $\Pi^*_I(b)$ is decreasing at $\frac{b^h_I}{2} \in (0, b^l_I]$, a contradiction. Thus, $b^I_l < \frac{b^I_h}{2}$. It follows from the implicit function theorem that $\frac{dc^I_\gamma}{d\gamma} = -\frac{{b^h_I}^2 - {b^l_I}^2}{\hat{\theta}^*_I(b^l_I, c^I_\gamma)} < 0$, where $\hat{\theta}^*_I(b^l_I, c^I_\gamma)$ is the cutoff type when $b = b^l_I$ and $c = c^I_\gamma$. The remainder of the lemma follows immediately. $\square$

Proof of Lemma 7

Proof. We consider three cases. First, if $b^*_D > \max\{\hat{b}, c^{R^{-1}}(c)\}$, then R-seeding occurs; thus, $\Pi^*_M(b^*_D) = c^I(b^*_D) - c - \gamma {b^*_D}^2$. The first-order condition of b is

$$\Pi^{*\prime}_M(b) = 2 - \frac{3}{2}\sqrt{1-b} - 2\gamma b = 0. \tag{A.9}$$

It can be shown that if $\gamma < \frac{4+\sqrt{7}}{8}$, then ${\Pi_M^*}'(b) > 0$ on $[0, \bar{b}]$; thus $b_D^* = \bar{b} := b_D^h$. If $\gamma \geq 1$, then b_D^* is given by the unique root of (A.9), $b_D^r := \frac{32\gamma - 9 - 3\sqrt{64\gamma^2 - 64\gamma + 9}}{32\gamma^2}$. It can be verified that b_D^r is decreasing in γ. Then, by the envelope theorem, $\frac{d\Pi_M^*(b_D^r)}{d\gamma} = -(b_D^r(\gamma))^2 \in (-1, 0)$ and $\frac{d^2\Pi_M^*(b_D^r)}{d\gamma^2} = -2b_D^r(\gamma){b_D^r}'(\gamma) > 0$. Note that $\Pi_M^*(b_D^h) \approx 1 - c - \gamma$. Thus, $\Pi_M^*(b_D^h)$ is single-crossing $\Pi_M^*(b_D^r)$ from above at some cutoff of γ such that $b_D^* = b_D^h$ if γ is less than this cutoff and $b_D^* = b_D^r$ otherwise. It follows that $\Pi_M^*(b_D^*)$ is decreasing and convex in γ.

Second, if ${c^M}^{-1}(c) < b_D^* \leq \hat{b}$, then M-seeding occurs; thus, $\Pi_M^*(b) = \frac{(1+b)^2}{4(2-b)} - c - \gamma b^2$. The first-order condition of b is

$$ {\Pi_M^*}'(b) = \frac{(1+b)(5-b)}{4(2-b)^2} - 2\gamma b = 0. \tag{A.10} $$

It can be shown that if γ is less than some cutoff $\tilde{\gamma} \approx 0.7456$, then ${\Pi_M^*}'(b) > 0$ on $[0, \bar{b}]$ and thus $b_D^* = \bar{b}$. If $\gamma > \tilde{\gamma}$, then b_D^* is given by the minimal root of (A.10), denoted b_D^m. By the envelope theorem, $\frac{d\Pi_M^*(b_D^m)}{d\gamma} = -(b_D^m(\gamma))^2 < 0$ and $\frac{d^2\Pi_M^*(b_D^m)}{d\gamma^2} = -2b_D^m(\gamma){b_D^m}'(\gamma) > 0$. Thus, $\Pi_M^*(b_D^m)$ is decreasing and convex in γ. Since $b_D^m < \hat{b}$, $\frac{d\Pi_M^*(b_D^m)}{d\gamma} > \frac{d\Pi_M^*(b)}{d\gamma}$ for all $b > \hat{b}$ such that R-seeding occurs. This implies that $\Pi_M^*(b_D^m)$ is single-crossing either $\Pi_M^*(b_D^h)$ or $\Pi_M^*(b_D^r)$ from below in terms of γ. It can be shown that $\Pi_M^*(b_D^m)$ intersects with $\Pi_M^*(b_D^h)$ at some cutoff $\hat{\gamma} \approx 0.826 \in (0.7456, \frac{4+\sqrt{7}}{8})$, such that $\Pi_M^*(b_D^m) < \Pi_M^*(b_D^h)$ if $\gamma < \hat{\gamma}$ and $\Pi_M^*(b_D^m) > \max\{\Pi_M^*(b_D^h), \Pi_M^*(b_D^r)\}$ if $\gamma > \hat{\gamma}$. Thus, when $\gamma < \hat{\gamma}$, if seeding occurs, then it is R-seeding and $b_D^* = b_D^h$; when $\gamma \geq \hat{\gamma}$, if seeding occurs, then it is M-seeding and $b_D^* = b_D^m$. Moreover, when M-seeding occurs, ${\Pi_M^*}'(0.5) < 0$, meaning that $b_D^m < 0.5$. Then, applying the implicit function theorem to (A.10), we have $\frac{db_D^m}{d\gamma} = -\frac{4b(2-b)}{4\gamma(2-3b)-1} < 0$ and $\frac{d^2 b_D^m}{d\gamma^2} = -\frac{8[2\gamma(2-b)^2 + 4\gamma b^2 - (1-b)]}{[4\gamma(2-3b)-1]^2} < 0$ since $0 < b_D^m < 0.5$ and $\gamma > \hat{\gamma}$.

Third, if $b_D^* < \min\{{c^R}^{-1}(c), {c^M}^{-1}(c)\}$, then seeding does not occur; thus, $\Pi_M^*(b_D^*) = \frac{(1-c)^2}{8(1-b_D^*)} - \gamma {b_D^*}^2$, where $b_D^* = b_D^l$ as characterized in Lemma 5. By Lemma 3, we have that the manufacturer prefers positive seeding to no seeding for c close to 0 and the other way around for c close to 1. Moreover, by Lemma 5, $\frac{d\Pi_M^*(b_D^l)}{dc} > -1 = \frac{d\Pi_M^*(b_D^h)}{dc} = \frac{d\Pi_M^*(b_D^m)}{dc}$. This implies that when $\gamma < \hat{\gamma}$, there exists a cutoff $c_\gamma^R \in (0, 1)$ such that $\Pi_M^*(b_D^h) > \Pi_M^*(b_D^l)$ and thus $b_D^* = b_D^h$ if $c < c_\gamma^R$, and $\Pi_M^*(b_D^h) \leq \Pi_M^*(b_D^l)$ and thus $b_D^* = b_D^l$ if $c \geq c_\gamma^R$. Similarly, when $\gamma \geq \hat{\gamma}$, there exists a cutoff $c_\gamma^M \in (0, 1)$ such that $b_D^* = b_D^m$ if $c < c_\gamma^M$ and $b_D^* = b_D^l$ if $c \geq c_\gamma^M$. By the envelope theorem, we have $\frac{dc_\gamma^R}{d\gamma} = -\frac{{b_D^h}^2 - {b_D^l}^2}{\hat{\theta}_I^*(b_D^l, c_\gamma^R)} < 0$, where $\hat{\theta}_D^*(b_D^l, c_\gamma^R)$ is the cutoff type when $b = b_D^l$ and $c = c_\gamma^R$. Note that when $\gamma \geq \tilde{\gamma}^D$, for all $c \geq c^M(b_D^m)$, we have

$$\Pi_M^{*}{}'(b_D^m) < \frac{(1-c^M(b_D^m))^2}{8(1-b_D^m)^2} - 2\gamma b_D^m = 2 + \frac{9(1-b_D^m)}{4(2-b_D^m)} - 3\sqrt{\frac{2(1-b_D^m)}{2-b_D^m}} - 2\gamma b_D^m$$
$$< 2 + \frac{9(1-b_D^m)}{4(2-b_D^m)} - 3 \cdot \frac{2(1-b_D^m)}{2-b_D^m} - 2\gamma b_D^m < \frac{(1+b_D^m)(5-b_D^m)}{4(2-b_D^m)^2} - 2\gamma b_D^m = 0.$$

The first inequality is because $c \geq c^M(b_D^m)$; the second is because $2(1-b_D^m) < (2-b_D^m)$; and the third is because $b_D^m < 0.5$. Thus, $\Pi_M^{*}{}'(b_D^m) < 0$, meaning that $b_D^l < b_D^m$. It follows that $\frac{dc_\gamma^M}{d\gamma} = -\frac{b_D^{m\,2} - b_D^{l\;2}}{\hat{\theta}_I^*(b_D^l, c_\gamma^M)} < 0$, where $\hat{\theta}_D^*(b_D^l, c_\gamma^R)$ is the cutoff type when $b = b_D^l$ and $c = c_\gamma^M$. The remainder of the Lemma follows immediately. □

Proof of Proposition 2

Proof. Proposition 2 is a simple corollary of Lemmas 6 and 7. □

Proof of Proposition 3

Proof. That when $\gamma < \hat{\gamma}$ and $c < c_\gamma^R$, $b_D^* > b_I^*$ follows immediately from Lemmas 6 and 7. The remainder has three parts. First, we claim that $b_D^l < b_I^l$. To show this, compare (A.5) with (A.7). Note that $b(1-b)^2$ is increasing and concave on $[0, \frac{1}{3}]$. Since b_l^D, $b_l^I < \frac{1}{3}$, it is easy to show that given c and γ, $b_D^l < \frac{b_I^l}{2}$. Second, we claim that $b_h^D < \frac{5b_h^I}{6}$. To show this, substituting $\frac{5b_h^I}{6}$ into $\Pi_M^m{}'(b)$, we have $\Pi_M^m{}'\left(\frac{5b_h^I}{6}\right) = \frac{(24\gamma-1)(120\gamma-35)}{4(48\gamma-17)^2} - \frac{10\gamma}{24\gamma-6}$. It follows that the RHS is negative for all $\gamma > 0.8$. Since $\gamma > \hat{\gamma}$, we have $b_h^D < \frac{5b_h^I}{6}$. Third, we claim that for all $\gamma > \hat{\gamma}$, $c_\gamma^M < c_\gamma^I$. To see this, note that $c_\gamma^{M}{}'(\gamma) = -\frac{b_D^{m\,2} - b_D^{l\;2}}{\hat{\theta}_I^*(b_D^l, c_\gamma^M)}$ and $c_\gamma^{I}{}'(\gamma) = -\frac{b_I^{h\,2} - b_I^{l\;2}}{\hat{\theta}_I^*(b_I^l, c_\gamma^I)}$. Since $b_l^I < \frac{b_h^I}{2}$ and $b_h^D < \frac{5b_h^I}{6}$, the difference between the numerators equals

$$(b_D^{m2} - b_D^{l\;2}) - (b_I^{h\,2} - b_I^{l\;2}) < b_D^{m2} + b_I^{l\;2} - b_I^{h2} < \left(\frac{5b_I^h}{6}\right)^2 + \left(\frac{b_I^h}{2}\right)^2 - b_I^{h\,2} = -\frac{b_I^{h\,2}}{18} < 0.$$

Moreover, the difference between the denominators is given by

$$\frac{3 + c_\gamma^M - 4b_D^l}{4(1-b_D^l)} - \frac{1 + c_\gamma^I - 2b_I^l}{2(1-b_I^l)} > \frac{3 + c_\gamma^M - 4b_I^l}{2(1-b_I^l)} - \frac{1 + c_\gamma^I - 2b_I^l}{2(1-b_I^l)} > \frac{c_\gamma^M - c_\gamma^I}{2(1-b_I^l)}. \tag{A.11}$$

The first inequality is due to that $b_D^l < b_I^l$ and the first term is decreasing in b; the second is due to that $\hat{\theta}_D^* > \hat{\theta}_I^*$ when seeding occurs in neither case. Since $b_I^l, b_D^l \to 0$ as $\gamma \to \infty$, by the definitions of c_γ^M and c_γ^I, we have that c_γ^M, $c_\gamma^I \to 0$, as $\gamma \to \infty$. This implies that the difference between the denominators is positive for sufficiently large γ. It follows that ${c_\gamma^M}'(\gamma) < {c_\gamma^I}'(\gamma)$, and thus, $c_\gamma^M < c_\gamma^I$ for sufficiently large γ. Suppose there exists a $\gamma' > \hat{\gamma}$ such that $c_\gamma^M = c_\gamma^I$, then we must have ${c_\gamma^M}'(\gamma') > {c_\gamma^I}'(\gamma')$. But then the RHS of (A.11) equals 0, i.e., the difference between the denominators is still positive. Thus, ${c_\gamma^M}'(\gamma') < {c_\gamma^I}'(\gamma')$, a contradiction. Then, by the continuity of c_γ^M and c_γ^I, for all $\gamma > \hat{\gamma}$, $c_\gamma^M < c_\gamma^I$. It follows that when $\gamma \geq \hat{\gamma}$, if $c < c_\gamma^I$, then b_D^* equals either b_D^m or b_D^l, while $b_I^* = b_I^h$, and thus, $b^D < b^I$. If $c \geq c_\gamma^I$, then $b_D^* = b_D^l < b_I^l = b_I^*$. Lastly, when $\gamma < \hat{\gamma}$ and $c \geq c_\gamma^R$, $b_D^* = b_D^l$, while b_I^* equals either b_I^h or b_I^l. Since $b_D^l < b_I^l < b_I^h$, we have $b_D^* < b_I^*$. □

Proof of Proposition 4

Proof. Proposition 4 is a simple corollary of Lemmas 1, 2 and 3. □

Proof of Proposition 5

Proof. Let $\Pi_D^* = \Pi_M^* + \Pi_R^*$. It follows that $\frac{\partial L}{\partial \gamma}$ has the same sign as $\Pi_D^* {\Pi_I^*}' - \Pi_I^* {\Pi_D^*}'$. We consider two cases. First, if $\gamma < \tilde{\gamma}^D$ and $c < \min\{c_\gamma^I, c_\gamma^D\}$, then $b_I^* = \frac{1}{2\gamma}$ and $b_D^* = \bar{b}$. It can be shown that in this case $\Pi_D^* {\Pi_I^*}' - \Pi_I^* {\Pi_D^*}' = \frac{(2\gamma-1)[1-c(1+2\gamma)]}{4\gamma^2} > 0$. The inequality is because $\gamma > \frac{1}{2}$ and $\Pi_I^* = \frac{1}{4\gamma} - c > 0$. Thus, the efficiency loss is increasing in γ. Second, if $\gamma \geq \tilde{\gamma}^I$, then by Lemmas 4 and 5, $b_I^* = b_I^l$ and $b_D^* = b_D^l$. By the envelope theorem, we have

$$\Pi_D^* {\Pi_I^*}' - \Pi_I^* {\Pi_D^*}' = \frac{(1-c)^2[4{b_D^l}^2(1-b_D^l) - 3{b_I^l}^2(1-b_I^l)]}{16(1-b_I^l)(1-b_D^l)}.$$

By the definition of b_I^l and b_D^l, we have $b_I^l > b_D^l$ and $b_I^l(1-b_I^l)^2 = 2b_D^l(1-b_D^l)^2$. Substituting this equation into above, $\Pi_D^* {\Pi_I^*}' - \Pi_I^* {\Pi_D^*}'$ has the same sign as $\frac{2(1-b_I^l)b_D^l}{1-b_D^l} - 3b_I^l$, which is negative since $b_I^l > b_D^l$. Thus, the efficiency loss is decreasing in γ. □

Proof of Lemma 8

Proof. Given W, we have $\frac{\partial W}{\partial b} = (1-\underline{\theta})^2 - 2\gamma b$ and $\frac{\partial W}{\partial \underline{\theta}} = (2b-1)\underline{\theta} - (2b-c)$. We claim that $b_S^* \leq \frac{1}{2\gamma}$. Suppose not, then $b_S^* > \frac{1}{2\gamma}$. Since $(1-\underline{\theta})^2 \leq 1$, $\frac{\partial W(b_S^*)}{\partial b} < 0$, a contradiction. It is easy to show that $\underline{\theta}_S^* = 0$ if and only if $b_S^* = b_S^h := \frac{1}{2\gamma}$. Moreover, if $\underline{\theta}_S^* = 0$, then $c \leq \frac{1}{\gamma}$. Thus, if $c > \frac{1}{\gamma}$, we have $\underline{\theta}_S^* > 0$. Then, by the first-order condition of $\underline{\theta}$, we have $b_S^* < \frac{1}{2}$. It follows that $b_S^* = \frac{1}{2\gamma}\left(\frac{1-c}{1-2b_S^*}\right)^2$ and $\underline{\theta}_S^* = \frac{c-2b_S^*}{1-2b_S^*}$. Substituting $\underline{\theta} = \frac{c-2b}{1-2b}$ into $\frac{\partial W}{\partial b}$, we have

$$\frac{\partial W}{\partial b} = \frac{2\gamma}{(1-2b)^2}\left[\frac{(1-c)^2}{2\gamma} - b(1-2b)^2\right]. \tag{A.12}$$

Note that $b(1-2b)^2$ is single-peaked on $[0, \frac{1}{2}]$ and is (locally) maximized at $b = \frac{1}{6}$, with a maximum equal to $\frac{2}{27}$. Thus, (A.12) has a root on $[0, \frac{1}{2}]$ if and only if $c \geq 1 - \frac{2}{3}\sqrt{\frac{\gamma}{3}}$, which is less than $\frac{1}{\gamma}$. Thus, when $c > \frac{1}{\gamma}$, (A.12) has two roots on $(0, \frac{1}{2})$. In particular, b_S^* is given by the smaller root, denoted b_S^l, such that $b_S^l < \frac{1}{6}$. Then, by the implicit function theorem, we have $\frac{db_S^l}{dc} = -\frac{1-c}{\gamma(1-2b_S^l)(1-6b_S^l)} < 0$ since $b_S^l < \frac{1}{6}$. It follows that ${b_S^l}''(c)$ has the same sign as $(1-2b_S^l)(1-6b_S^l) - (1-c)(8-24b_S^l){b_S^l}'(c)$, which is positive since ${b_S^l}'(c) < 0$ and $b_S^l < \frac{1}{6}$. Thus, ${b_S^l}''(c) > 0$. By the envelope theorem, we have $\frac{dW(b_S^l)}{dc} = \underline{\theta} - 1 \in (-1, 0)$, and $\frac{d^2W(b_S^l)}{dc^2} = \frac{1-6b_S^l+2c-2(1-2b_S^l){b_S^l}'(c)}{(1-2b_S^l)^2} > 0$ since ${b_S^l}'(c) < 0$ and $b_S^l < \frac{1}{6}$. Thus, $W(b_S^l)$ is decreasing and convex in c. It is also easy to show that $W(b_S^l) \to 0$ as $c \to 1$.

Then, consider when $c \leq \frac{1}{\gamma}$. There are two cases. First, suppose $\gamma < 3$. If $c < 1 - \frac{2}{3}\sqrt{\frac{\gamma}{3}}$, then (A.12) has no root, meaning that $W(b_S^h) > W(b_S^l)$. Since $W(b_S^h) = \frac{1}{4\gamma} - c + \frac{1}{2}$, we have $\frac{dW(b_S^h)}{dc} = -1$, and $W(b_S^h) < 0$ for c close to 1. It follows that $W(b_S^h)$ intersects with $W(b_S^l)$ from above at some $c_\gamma^S \in (0, 1)$ in terms of c. In addition, note that if $c = \frac{1}{\gamma}$, then $\gamma > 1$ and (A.12) can be rewritten as

$$\frac{\partial W}{\partial b} = -\frac{2\gamma}{(1-2b)^2}\left(b - \frac{1}{2\gamma}\right)\left(b - \frac{1}{2} + \frac{1+\sqrt{4\gamma-3}}{4\gamma}\right)\left(b - \frac{1}{2} - \frac{1+\sqrt{4\gamma-3}}{4\gamma}\right). \quad \text{(A.13)}$$

It follows that when $\gamma < 3$, $b_S^l = \frac{1}{2} - \frac{1+\sqrt{4\gamma-3}}{4\gamma} < b_S^h$, and $W'(b) \geq 0$ on $[0, b_S^l]$ and $W'(b) \leq 0$ on $[b_S^l, b_S^h]$. Thus, when $c = \frac{1}{\gamma}$, $W(b_S^h) < W(b_S^l)$. This implies that $c_\gamma^S < \frac{1}{\gamma}$. In summary, when $\gamma < 3$, $b_S^* = b_S^h$ and $\underline{\theta}_S^* = 0$ if $c < c_\gamma^S$, and $b_S^* = b_S^l$ and $\underline{\theta}_S^* = \frac{c-2b_S^l}{1-2b_S^l}$ otherwise. By the envelope theorem, we have $\frac{dc_\gamma^S}{d\gamma} = -\frac{{b_S^h}^2 - {b_S^l}^2}{\underline{\theta}_S^*(b_S^l)} < 0$. Second, suppose $\gamma \geq 3$. (A.13) implies that when $c = \frac{1}{\gamma}$, $b_S^l = \frac{1}{2\gamma} = b_S^h$. It follows that when $\gamma \geq 3$ and $c = \frac{1}{\gamma}$, $b_S^* = b_S^h$. Next, consider $c < \frac{1}{\gamma}$. Suppose $b_S^* = b_S^l$. Since ${b_S^l}'(c) < 0$, we have $b_S^l(c) > b_S^l(\frac{1}{\gamma}) = \frac{1}{2\gamma}$. This leads to a contradiction since $b_S^* \leq \frac{1}{2\gamma}$. Thus, $b_S^* = b_S^h$. In summary, when $\gamma \geq 3$, $b_S^* = b_S^h$ if $c < \frac{1}{\gamma}$ and $b_S^* = b_S^l$ otherwise; thus, $c_\gamma^S = \frac{1}{\gamma}$ and b_S^* is continuous in c on $(0, 1)$. It follows from the continuity of W that c_γ^S is continuous at $\gamma = 3$. Indeed, it can be shown that if $\frac{1}{2} < \gamma < 3$, then $c_\gamma^S = \frac{4-\sqrt{6\gamma-2}}{3} + \frac{(\sqrt{6\gamma-2}-1)^3}{27\gamma}$; if $\gamma \geq 3$, $c_\gamma^S = \frac{1}{\gamma}$. Thus, c_γ^S decreases with γ. □

Proof of Corollary 3

Proof. By Lemmas 6 and 8, we have $b_I^h < b_S^h$ and it is also easy to show that $b_I^l < b_S^l$. By Lemma 6, we have $c_\gamma^I \leq c^I(b_I^h) = \left(\frac{4\gamma-2}{4\gamma-1}\right)^{\frac{3}{2}} - \frac{4\gamma-3}{4\gamma-1}$. It follows from the proof of Lemma 8 that $c_\gamma^I < c_\gamma^S$ for all $\gamma > \frac{1}{2}$. Since $b_I^l < b_I^h$, if $c < c_\gamma^I$, then $b_I^* = b_I^h < b_S^h = b_S^*$; if $c \in [c_\gamma^I, c_\gamma^S)$, then $b_I^* = b_I^l < b_S^h = b_S^*$; if $c \geq c_\gamma^S$, then $b_I^* = b_I^l < b_S^l = b_S^*$. □

Proof of Proposition 6

Proof. By Lemma 8 and Corollary 3, when $c < \min\{c_\gamma^I, c_\gamma^R\} < c_\gamma^S$, $b_S^* = \frac{1}{2\gamma}$ and $\underline{\theta}_S^* = 0$. In addition, by Lemmas 6 and 7, $b_I^* = \frac{1}{4\gamma-1}$ and $\hat{\theta}_I^* = 0$, and $b_D^* = \bar{b}$ and $\hat{\theta}_D^* = 0$. It follows that $|b_D^* - b_S^*| < |b_S^* - b_I^*|$ if and only if $\frac{3+\bar{b}-\sqrt{\bar{b}^2-10\bar{b}+9}}{8\bar{b}} < \gamma < \frac{3+\bar{b}+\sqrt{\bar{b}^2-10\bar{b}+9}}{8\bar{b}}$. It is easy to verify that the LHS is less than $\frac{1}{2}$ given that $\bar{b} < 1$, and the RHS is less than $\hat{\gamma}$ for $\bar{b}$ close to 1, meaning that R-seeding indeed occurs in the decentralized case. Since W is strictly concave in b and is maximized at $b_S^* = \frac{1}{2\gamma}$, the decentralized case yields higher social welfare if $\frac{1}{2} < \gamma < \frac{3+\bar{b}+\sqrt{\bar{b}^2-10\bar{b}+9}}{8\bar{b}}$. □

C Costly Targeting

In this section, as an extension, we consider when firms incur additional costs to seed the customers, for example the relevant marketing costs. Specifically, we assume that each of the central planner and the value chain participants incurs a unit seeding cost $k > 0$ on top of the cost of the product c and w, respectively. The analysis is analogous to that in the main body. Thus, we only present relevant results here.

C.1 Exogenous Network Effects

We first consider when $b \in (0,1)$ is fixed. We start with the centralized case. The central planner's profit is given by $\Pi_I = [(1-b)\hat{\theta} + (1+\alpha)b - c](1-\hat{\theta}) - (c+k)\alpha$. The lemma below characterizes the central planner's optimal strategy.

Lemma A3. *With seeding cost k, the central planner's optimal strategy satisfies that*

i. When $c < c_k^I$, $\alpha_I^* = \hat{\theta}_I^* = \max\{0, \frac{1-k-b}{2}\}$, thus, $\Pi_I^* = \hat{\theta}_I^{*2} + b - c$;

ii. When $c \geq c_k^I$, $\alpha_I^* = 0$, $\hat{\theta}_I^* = \frac{1+c-2b}{2(1-b)}$, thus, $\Pi_I^* = \frac{(1-c)^2}{4(1-b)}$,

where the cutoff $c_k^I := \max\{0, 1-2(1-b) + (1-k-b)\sqrt{1-b}, 2b-1\}$.

Lemma A3 indicates that the results of Lemma 2 are robust to small perturbation of k. As it becomes more costly, seeding occurs less often in the centralized case, i.e., $c_k^I < c^I$ for all $b \in (0,1)$ and $k > 0$. In particular, when b is sufficiently low, seeding never occurs no matter how small c is. On the other hand, when b is sufficiently high (i.e., $b > 1-k$), the market is fully covered without seeding. Intuitively, with strong network effects, the central planner would like to expand the user network as much as possible. Since seeding is now more costly, the central planner finds it more profitable to penetrate the market by charging a low price than by seeding the customers.

Then, we turn to the decentralized case. The retailer's and manufacturer's profits are $\Pi_R = [(1-b)\hat{\theta}+(1+\alpha_M+\alpha_R)b-w](1-\hat{\theta})-(w+k)\alpha_R$ and $\Pi_M = (w-c)(1-\hat{\theta}+\alpha_R)-(c+k)\alpha_M$, respectively, subject to $0 \le \alpha_M + \alpha_R \le \hat{\theta} \le 1$. The following lemma characterizes the equilibrium outcome of the decentralized case.

Lemma A4. *With seeding cost k, the equilibrium of the decentralized case satisfies that there exist cutoffs $c_k^R := \max\{0, 1-4(1-b)+\sqrt{8(1-k-b)(1-b)^{1.5}}, 4b-3\}$ and $c_k^M := \max\{0, 1-4(1-b)+[3(1-b)-k]\sqrt{\frac{2(1-b)}{2-b}}, 4b-3\}$ and $\hat{k} \approx 0.07$ such that*

1. *If $k < \hat{k}$, let $\hat{b}_k^l < \hat{b}_k^h$ be the two elements of $\{b \in (0,1) | 0 < c^R = c^M < 4b-3\}$, then*
   - *(a) When $b \in (1-\frac{k}{3}, 1) \cup (\hat{b}_k^l, \hat{b}_k^h]$ and $c < c_k^R$, $\alpha_M^* = 0$, $w^* = c_k^I$, $\alpha_R^* = \hat{\theta}_D^* = \max\{0, \frac{1-k-b}{2}\}$, thus, $\Pi_M^* = c_k^I - c$, $\Pi_R^* = \hat{\theta}_D^{*2} + b - c_k^I$;*
   - *(b) When $b \in (\hat{b}_k^h, 1-\frac{k}{3}] \cup (0, \hat{b}_k^l]$ and $c < c_k^M$, $\alpha_M^* = \frac{3(1-b)-k}{2(2-b)}$, $w^* = \frac{1+b-k}{2}$, $\alpha_R^* = 0$, $\hat{\theta}_D^* = \frac{3(1-b)-k}{2(2-b)}$, thus, $\Pi_M^* = \frac{(1+b)^2-6(1-b)k+k^2}{4(2-b)} - c$, $\Pi_R^* = \frac{(1-b)(1+b+k)^2}{4(2-b)^2}$;*
   - *(c) When $c \ge \max\{c_k^R, c_k^M\}$, $\alpha_M^* = 0$, $w_D^* = \frac{1+c}{2}$, $\alpha_R^* = 0$, $\hat{\theta}_D^* = \frac{3+c-4b}{4(1-b)}$, thus, $\Pi_M^* = \frac{(1-c)^2}{8(1-b)}$, $\Pi_R^* = \frac{(1-c)^2}{16(1-b)}$;*
2. *If $\hat{k} \le k < \frac{3}{4}$, then the equilibrium outcome is given by (1.a) when $b > 1-\frac{k}{3}$ and $c < c_k^R$, by (1.b) when $b \le 1-\frac{k}{3}$ and $c < c_k^M$, and by (1.c) when $c \ge \max\{c_k^R, c_k^M\}$.*
3. *If $k \ge \frac{3}{4}$, then $c_k^R = c_k^M = \max\{0, 4b-3\}$ and thus the equilibrium outcome is given by (1.a) if $c < c_k^R = c_k^M$, and by (1.c) otherwise.*

Lemma A4 indicates that the results of Lemma 3 are robust to small perturbation of k. Similarly, seeding is less often in the decentralized case due to the positive seeding cost. As in the centralized case, seeding never occurs for both sufficiently low and sufficiently high b. In particular, when $b > 1-\frac{k}{3}$, the market is also fully covered without seeding. As k rises, whereas both R-seeding and M-seeding become less likely (i.e., both c_k^R and c_k^M move downward), R-seeding occurs relatively less often. Specifically, for small seeding cost $k < \hat{k}$, R-seeding only occurs for intermediate levels of $b \in (\hat{b}_k^l, \hat{b}_k^h]$, which is a subset of the R-seeding region $(\hat{b}, 1)$ when $k = 0$. Moreover, for relatively high seeding cost $k \ge \hat{k}$, R-seeding never occurs, whereas M-seeding remains so long as $k < \frac{3}{4}$ and seeding disappears in equilibrium if $k \ge \frac{3}{4}$. Intuitively, to induce R-seeding, the manufacturer has to charge a relatively low wholesale price c_k^I. Alternatively, it can charge a higher price $\frac{1+b-k}{2}$ and seed the customers itself. As the seeding cost rises, it turns out that the lower price c_k^I reduces relatively faster, rendering R-seeding less attractive to the manufacturer.

C.2 Endogenous Network Effects

Now, we turn to the case where firms can engineer the network effect. To ensure that each seeding firm can occur under certain conditions, we assume that $k > 0$ is sufficiently low. We again start with the centralized case. The central planner's profit is given by

$$\Pi_I = \begin{cases} b - c - \gamma b^2 & \text{if } b > 1 - k \text{ and } c < c_k^I; \\ \frac{(1-k-b)^2}{4} + b - c - \gamma b^2 & \text{if } b \leq 1 - k \text{ and } c < c_k^I; \\ \frac{(1-c)^2}{4(1-b)} - \gamma b^2 & \text{if } c \geq c_k^I. \end{cases}$$

The lemma below characterizes the central planner's optimal strategy.

Lemma A5. *In the generalized model, the central planner's optimal strategy satisfies that*

1. *If $\gamma \in (\frac{1}{2}, \frac{1}{2(1-k)})$, then there exists a cutoff $\hat{c}_{\gamma k}^I \in (0, c_\gamma^I)$ such that*
   - *(a) When $c < \hat{c}_{\gamma k}^I$, $b_I^* = \hat{b}_I^h := \frac{1}{2\gamma}$, $\alpha_I^* = \hat{\theta}_I^* = 0$, $\Pi_I^*(\hat{b}_I^h) = \frac{1}{4\gamma} - c$;*
   - *(b) When $c \geq \hat{c}_{\gamma k}^I$, $b_I^* = b_I^l \in (0, \frac{\hat{b}_I^h}{2})$, which is the minimal root of*

   $$b(1-b)^2 = \frac{(1-c)^2}{8\gamma},$$

   with $b_I^{l\prime}(c) < 0$ and $b_I^{l\prime\prime}(c) > 0$, $\alpha_I^* = 0$, $\hat{\theta}_I^* = \frac{1-2b_I^l+c}{2(1-b_I^l)}$, $\Pi_I^*(b_I^l) = \frac{(1-c)^2}{4(1-b_I^l)} - \gamma {b_I^l}^2$,

   where $\hat{c}_{\gamma k}^I$ is the unique root of $\Pi_I^*(\hat{b}_I^h) = \Pi_I^*(b_I^l)$ and is decreasing in γ;

2. *If $\gamma > \frac{1}{2(1-k)}$, then there exists a cutoff $\tilde{c}_{\gamma k}^I \in (0, c_\gamma^I)$ such that*
   - *(a) When $c < \tilde{c}_{\gamma k}^I$, $b_I^* = \tilde{b}_I^h := \frac{1+k}{4\gamma-1}$, $\alpha_I^* = \hat{\theta}_I^* = \frac{2\gamma-1-2\gamma k}{4\gamma-1}$, $\Pi_I^*(\tilde{b}_I^h) = \frac{(1+k)^2}{4(4\gamma-1)} + \frac{(1-k)^2}{4} - c$;*
   - *(b) When $c \geq \tilde{c}_{\gamma k}^I$, the equilibrium outcome is given by (1.b),*

   where $\tilde{c}_{\gamma k}^I$ is the unique root of $\Pi_I^*(\tilde{b}_I^h) = \Pi_I^*(b_I^l)$ and is decreasing in γ.

Lemma A5 indicates that the results of Lemma 6 are also robust to small perturbation of k. Note that when the central planner is cost-efficient in engineering the network effect, i.e., $\gamma < \frac{1}{2(1-k)}$, it will choose a sufficiently high $b = \frac{1}{2\gamma}$ when it is also cost-efficient in production, i.e, $c < \hat{c}_{\gamma k}^I$, such that the market is fully covered without seeding. Thus, the equilibrium coincides with that of Lemma 4. In contrast, when the central planner is less efficient in engineering the network effect, i.e., $\gamma \geq \frac{1}{2(1-k)}$, it will choose a lower $b = \frac{1+k}{4\gamma-1}$ when it is cost-efficient in production, i.e, $c < \tilde{c}_{\gamma k}^I$, such that seeding occurs in equilibrium. Note too that $b = \frac{1+k}{4\gamma-1}$ is increasing in k. Intuitively, since the marginal cost of seeding

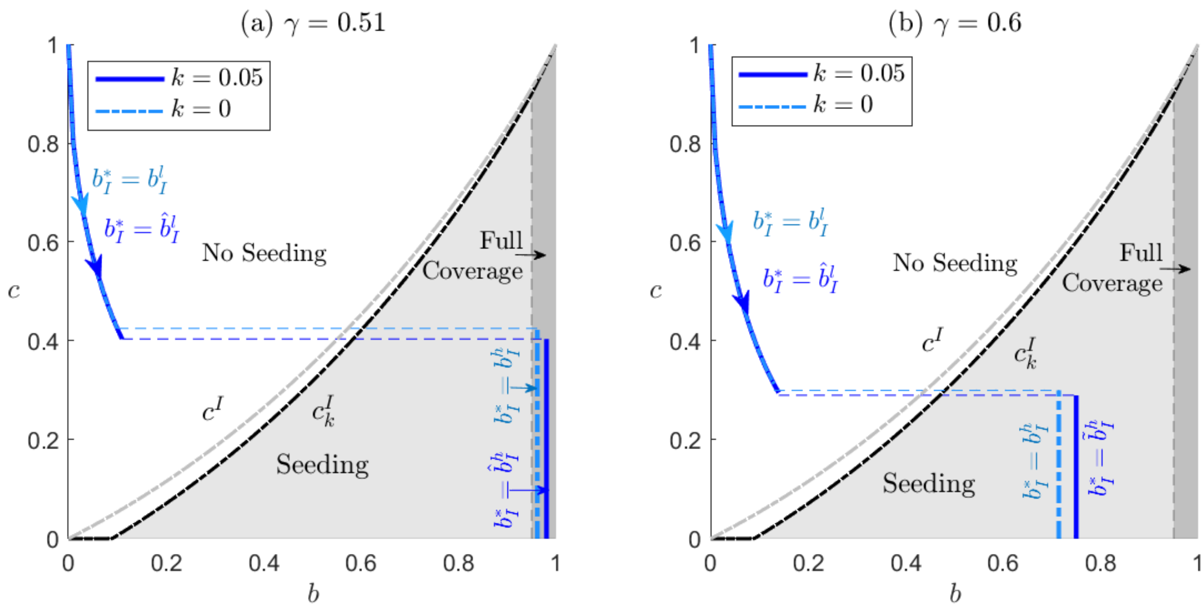

Figure 6: The Equilibrium of the Centralized Case

$c + k$ is increasing in k, the marginal benefit of seeding must be higher correspondingly as k increases, leading to a higher degreed of network effect. This is illustrated by Figure 6.

Lastly, we turn to the decentralized case. The manufacturer's profit is given by

$$\Pi_M = \begin{cases} c_k^I - c - \gamma b^2 & \text{if } b \in (1 - \frac{k}{3}, 1) \cup (\hat{b}_k^l, \hat{b}_k^h] \text{ and } c < c_k^R; \\ \frac{(1+b)^2 - 6(1-b)k + k^2}{4(2-b)} - c - \gamma b^2 & \text{if } b \in (\hat{b}_k^h, 1 - \frac{k}{3}] \cup (0, \hat{b}_k^l] \text{ and } c < c_k^M; \\ \frac{(1-c)^2}{8(1-b)} - \gamma b^2 & \text{if } c \geq \max\{c_k^R, c_k^M\}. \end{cases}$$

The next lemma characterizes the equilibrium outcome of the decentralized case.

Lemma A6. *In the generalized model, the equilibrium of the decentralized case satisfies that there exist cutoffs $\hat{\gamma}_k$, and $c_{\gamma k}^R, c_{\gamma k}^M > 0$ such that*

i. When $\gamma < \hat{\gamma}_k$ and $c < c_{\gamma k}^R$, $b_D^* = b_D^h := \bar{b}$, $\alpha_M^* = 0$, $w^* = 2b_D^h - 1$, $\alpha_R^* = \hat{\theta}_D^* = 0$, $\Pi_M^*(b_D^h) = 2b_D^h - 1 - c - \gamma {b_D^h}^2$, $\Pi_R^*(b_D^h) = 1 - b_D^h$;

ii. When $\gamma \geq \hat{\gamma}_k$ and $c < c_{\gamma k}^M$, $b_D^* = b_D^m \in (0, b_D^h)$, which is the minimal root of

$$8\gamma b^3 - (32\gamma - 1)b^2 + (32\gamma - 4)b - (k^2 + 6k + 5) = 0,$$

with $b_D^{m\prime}(\gamma) < 0$ and $b_D^{m\prime\prime}(\gamma) < 0$, $\alpha_M^* = \frac{3(1-b_D^m)-k}{2(2-b_D^m)}$, $w^* = \frac{1+b_D^m-k}{2}$, $\alpha_R^* = 0$, $\hat{\theta}_D^* = \frac{3(1-b_D^m)-k}{2(2-b_D^m)}$, $\Pi_M^*(b_D^m) = \frac{(1+b_D^m)^2-6(1-b_D^m)k+k^2}{4(2-b_D^m)} - c - \gamma {b_D^m}^2$, $\Pi_R^*(b_D^m) = \frac{(1-b_D^m)(1+b_D^m+k)^2}{4(2-b_D^m)^2}$;

iii. When $c \geq \max\{c_{\gamma k}^R, c_{\gamma k}^M\}$, $b_D^* = b_D^l \in (0, b_D^m)$, which is the minimal root of

$$b(1-b)^2 = \frac{(1-c)^2}{16\gamma},$$

with ${b_D^l}'(c) < 0$ *and* ${b_D^l}''(c) > 0$, $\alpha_M^* = 0$, $w^* = \frac{1+c}{2}$, $\alpha_R^* = 0$, $\hat{\theta}_D^* = \frac{3+c-4b_D^l}{4(1-b_D^l)}$, $\Pi_M^*(b_D^l) = \frac{(1-c)^2}{8(1-b_D^l)} - \gamma {b_D^l}^2$, $\Pi_R^*(b_l^D) = \frac{(1-c)^2}{16(1-b_l^D)}$.

Moreover, when $\gamma < \hat{\gamma}_k$, $c_{\gamma k}^R$ *is the root of* $\Pi_M^*(b_D^h) = \Pi_M^*(b_D^l)$ *and is decreasing in* γ; *when* $\gamma \geq \hat{\gamma}_k$, $c_{\gamma k}^M$ *is the root of* $\Pi_M^*(b_D^m) = \Pi_M^*(b_D^l)$ *and is decreasing in* γ.

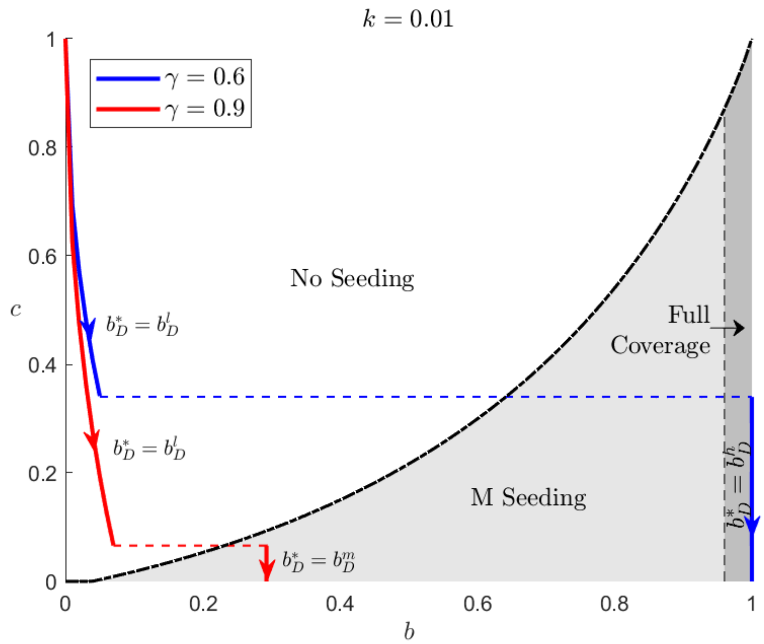

Figure 7: The Equilibrium of the Decentralized Case

Lemma A6 indicates that the results of Lemma 7 are robust to small perturbation of k. The difference is that when the manufacturer is cost-efficient in both engineering the network effect and production, i.e., $\gamma < \hat{\gamma}_k$ and $c < c_{\gamma k}^R$, it will again choose the highest level $\bar{b}$. However, in contrast to when $k = 0$ such that R-seeding occurs, the market is fully covered without seeding (see Figure 7), since at such a high level of b the retailer finds it more profitable to penetrate the market by charging a low price than by seeding the customer.

D Uniform Seeding

In this section, we consider when firms have lower targeting capabilities in seeding. In particular, we focus on the case in which neither the central planner nor the channel participants can target customers; instead, they can only uniformly distribute the seeds among all customer types, which is referred to as *uniform seeding*. Proposition A1 below characterizes the equilibrium seeding strategies under uniform seeding.

Proposition A1. *Under uniform seeding, the equilibrium seeding level is always zero in both the centralized and decentralized cases.*

Proposition A1 states that under uniform seeding, irrespective of the market structure, seeding never occurs in equilibrium. While the proof of Proposition A1 is somewhat complex and can be found below, the intuition is transparent. Whereas the explicit production cost might be negligible, there is always a nontrivial *implicit cost of cannibalization*. Specifically, with extra Δ units of seeding level, $(1-\hat{\theta})\Delta$ of higher value customers who would have purchased the product under the initial seeding strategy now receive seeds, thereby leading to a $p(1-\hat{\theta})\Delta$ loss of revenue. Thus, the *virtual* marginal cost of uniform seeding is the sum of the marginal cost and the cost of cannibalization.

It turns out that in the centralized case, the virtual marginal cost of seeding is always higher than the marginal benefit, which is the central planner's capability to charge a higher price due to the network effect. Thus, the central planner chooses not to seed. In the decentralized case, the retailer's marginal profit of seeding is the same as that of the central planner except for that the marginal cost is given by the wholesale price, and thus, the retailer chooses not to seed either. In contrast, the marginal benefit of seeding is lower for the manufacturer due to double marginalization. Specifically, the increase in final price caused by higher seeding level will only partially lead to the increase in manufacturer's revenue, as the retailer also has market power. Thus, the manufacturer has a lower incentive to seed the customers, thereby choosing not to seed.

Proof of Proposition A1

Under uniform seeding, the marginal type $\hat{\theta}$ is determined by the indifference condition: $U(\hat{\theta}; 1-\hat{\theta}+\alpha\hat{\theta}) = \hat{\theta} + b(1-\hat{\theta}+\alpha\hat{\theta}) - p = 0$. Thus, $p = (1-b+\alpha b)\hat{\theta} + b$. First, we consider the centralized case. Similarly, the central planner's profit is given by

$$\Pi_I := [(1+b\alpha-b)\hat{\theta}+b](1-\alpha)(1-\hat{\theta}) - (1-\hat{\theta}+\alpha\hat{\theta})c.$$

The lemma below suffices to prove the part of the centralized case of Proposition A1.

Lemma A7. *In the centralized case, (i) when $c < 2b-1$, $\alpha_I^* = 0$ and $\hat{\theta}_I^* = 0$; (ii) when $c \geq 2b-1$, $\alpha_I^* = 0$ and $\hat{\theta}_I^* = \frac{1+c-2b}{2(1-b)}$.*

Proof. Since $c < 1$, at the optimum $\hat{\theta} < 1$. Then, the marginal profit of α is given by

$$\frac{\partial \Pi_I}{\partial \alpha} = (1-\hat{\theta})[b\hat{\theta}(1-\alpha) - (1+b\alpha-b)\hat{\theta} - b] - \hat{\theta}c$$
$$< (1-\hat{\theta})[b\hat{\theta}(1-\alpha) - (1+b\alpha-b)\hat{\theta} - b\hat{\theta}] - \hat{\theta}c < 0.$$

Thus, for any pair (b, c), we have $\alpha_I^* = 0$. The remainder follows from Lemma 1. $\square$

The next lemma suffices to prove the part of the decentralized case of Proposition A1.

Lemma A8. *In the decentralized case, (i) when $c < 4b-3$, $\alpha_M^* = \alpha_R^* = 0$, $w^* = 2b-1$, $\hat{\theta}_D^* = 0$; (ii) when $c \geq 4b-3$, $\alpha_M^* = \alpha_R^* = 0$, $w^* = \frac{1+c}{2}$, $\hat{\theta}_D^* = \frac{3+c-4b}{4(1-b)}$.*

Proof. By backward induction, given (α_M, w), the retailer's profit is given by

$$\Pi_R = [(1+b\alpha-b)\hat{\theta}+b](1-\alpha)(1-\hat{\theta}) - [(1-\hat{\theta})(1-\alpha)+\alpha_R]w,$$

where $\alpha = \alpha_M + \alpha_R$. It follows that $\frac{\partial \Pi_R}{\partial \alpha_R} = (1-\hat{\theta})[b\hat{\theta}(1-\alpha) - (1+b\alpha-b)\hat{\theta} - b] - \hat{\theta}w$. By the same argument as in the proof of Lemma A7, $\frac{\partial \Pi_R}{\partial \alpha_R} < 0$. Thus, for any pair (b, w), $\alpha_R^* = 0$. Thus, the retailer's profit becomes $\Pi_R = [(1+b\alpha_M-b)\hat{\theta}+b-w](1-\alpha_M)(1-\hat{\theta})$. It follows that $\Pi_R{}'(\hat{\theta}) = (1-\alpha_M)[-2(1+b\alpha_M-b)\hat{\theta}+1+b\alpha_M-2b+w]$. Since $\hat{\theta} \in [0,1]$, the optimal marginal type $\hat{\theta}(\alpha_M, w)$ equals $\frac{1+w+b\alpha_M-2b}{2(1+b\alpha_M-b)}$ if $1+b\alpha_M \geq w > 2b-b\alpha_M-1$, and 0 if $w \leq 2b-b\alpha_M-1$. Then, by backward induction, the manufacturer's profit is given by

$$\Pi_M = \begin{cases} \frac{(1-\alpha_M)(1+b\alpha_M-w)(w-c)}{2(1+b\alpha_M-b)} - c\alpha_M & \text{if } 1+b\alpha_M \geq w > 2b-b\alpha_M-1; \\ (1-\alpha_M)(w-c) - c\alpha_M & \text{if } w \leq 2b-b\alpha_M-1. \end{cases}$$

Clearly, if $w \leq 2b-b\alpha_M-1$, then the manufacturer will choose $\alpha_M = 0$ and $w = 2b-1$, and thus, $\Pi_M = 2b-1-c$. If $1+b\alpha_M \geq w > 2b-b\alpha_M-1$, then the marginal profit of α_M is given by $\frac{\partial \Pi_M}{\partial \alpha_M} = \frac{(w-c)(-b^2\alpha_M^2+2b^2\alpha_M-2b\alpha_M-b^2+b-1+w)}{2(1+b\alpha_M-b)^2} - c$. It follows that $\frac{\partial^2 \Pi_M}{\partial \alpha_M^2} = \frac{b(w-c)(b-w)}{(1+b\alpha_M-b)^3}$. Without loss of generality, focus on $w > c$. Thus, when $b \leq c$, $\frac{\partial^2 \Pi_M}{\partial \alpha_M^2} < 0$. It follows that

$$\frac{\partial \Pi_M}{\partial \alpha_M} \leq \frac{\partial \Pi_M}{\partial \alpha_M}\bigg|_{\alpha_M=0} = \frac{(w-c)(-b^2+b-1+w)}{2(1-b)^2} - c \leq \frac{(w-c)(-b^2+b)}{2(1-b)^2} - c < 0.$$

The second inequality is because $w \leq 1$; the last inequality is because $b \leq c < w \leq 1$. Thus, when $b \leq c$, the manufacturer will choose $\alpha_M = 0$ if $b \leq c$. When $b > c$, $\frac{\partial^2 \Pi_M}{\partial \alpha_M^2} < 0$ if and only if $w > b$. The marginal profit of w is given by $\frac{\partial \Pi_M}{\partial w} = \frac{(1-\alpha_M)(-2w+1+c+b\alpha_M)}{2(1+b\alpha_M-b)}$. This implies that given α_M, the optimal w is equal to $\max\left\{\frac{1+c+b\alpha_M}{2}, 2b-b\alpha_M-1\right\}$. Recall that if $w = 2b-b\alpha_M-1$, then $\alpha_M = 0$. Thus, it remains to study when $w = \frac{1+c+b\alpha_M}{2}$. We consider two cases. First, if $b < \frac{1+c}{2}$, then $w \geq b$. It follows that

$$\frac{\partial \Pi_M}{\partial \alpha_M} \leq \frac{\partial \Pi_M}{\partial \alpha_M}\Big|_{\alpha_M=0} = \frac{(\frac{1+c}{2} - c)(-b^2 + b - 1 + \frac{1+c}{2})}{2(1-b)^2} - c.$$

Taking derivative with respect to b, we have $\frac{\partial}{\partial b}\frac{\partial \Pi_M}{\partial \alpha_M}\Big|_{\alpha_M=0} = \frac{-(1-c)(b-c)}{4(1-b)^3} < 0$, meaing that

$$\frac{\partial \Pi_M}{\partial \alpha_M}\Big|_{\alpha_M=0} \leq \frac{(\frac{1+c}{2} - c)(-1 + \frac{1+c}{2})}{2} - c = \frac{-(1-c)^2}{8} - c < 0.$$

Thus, if $b < \frac{1+c}{2}$, then the manufacturer will choose $\alpha_M = 0$. Finally, if $b \geq \frac{1+c}{2}$, then there exists a unique $\alpha_M \in (0, 1)$ such that $w = \frac{1+c+b\alpha_M}{2} = b$. Thus, $\frac{\partial \Pi_M}{\partial \alpha_M}$ is maximized when $b\alpha_M = 2b - 1 - c$, i.e., $w = b$. Substituting $w = b$, we have

$$\begin{aligned}\frac{\partial \Pi_M}{\partial \alpha_M} \leq \frac{\partial \Pi_M}{\partial \alpha_M}\Big|_{b\alpha_M=2b-1-c} &= \frac{(b-c)(-b^2\alpha_M^2 + 2b^2\alpha_M - 2b\alpha_M - b^2 + 2b - 1)}{2(1 + b\alpha_M - b)^2} - c \\ &= \frac{-(b-c)[b\alpha_M(1-c) + (1-b)^2]}{2(b-c)^2} - c < 0.\end{aligned}$$

The second equality is from substituting $b\alpha_M = 2b - 1 - c$. Thus, $\alpha_M^* = 0$. In summary, for any pair (b, c), we have $\alpha_R^* = \alpha_M^* = 0$. The remainder follows from Lemma 1. □

E Multiplicative Utility Function

In this section, we consider heterogeneous network effects with a multiplicative form utility function, $U(\theta; N) := \theta(1 + bN) - p$. This can be interpreted as that high-type customers obtain high benefits from the data network effect compared to low-type customers.

E.1 The Baseline Model

Similar to the main body, we start with a baseline model without seeding or engineering b. In the centralized case, the central planner chooses a marginal type $\hat{\theta}$ to maximize its profit $\Pi_I(\hat{\theta}) := [\theta(1 + b(1-\hat{\theta})) - c](1-\hat{\theta})$. In the decentralized case, the retailer's and manufacturer's profits are $\Pi_R(\hat{\theta}, w) := [\theta(1 + b(1-\hat{\theta})) - w](1-\hat{\theta})$, $\Pi_M(w) := (w - c)(1-\hat{\theta})$, respectively. The next lemma characterizes the equilibrium outcome of the baseline model.

Lemma A9. *There exists a unique equilibrium such that*

i. (centralized case) The central planner's optimal strategy satisfies that $\hat{\theta}_I^* = \frac{1+2b-\sqrt{b^2+(1-3c)b+1}}{3b}$, *thus* $\Pi_I^* = \frac{(b-1+\sqrt{b^2+(1-3c)b+1})(1+b^2+4b-6bc-(1-b)\sqrt{b^2+(1-3c)b+1})}{27b^2}$.

ii. (decentralized case) The equilibrium of the decentralized case satisfies that

$w^* = \frac{4+4b^2+10b+9bc-2(1-b)\sqrt{4b^2+(1-9c)b+4}}{27b}$, $\hat{\theta}^* = \frac{2+7b-\sqrt{4b^2+(1-9c)b+4}}{9b}$, *thus*

$\Pi_M = \frac{(4+4b^2+10b-18bc-2(1-b)\sqrt{4b^2+(1-9c)b+4})(\sqrt{4b^2+(1-9c)b+4}-2(1-b))}{243b^2}$,

$\Pi_R = \frac{(2-2b-\sqrt{4b^2+(1-9c)b+4})(2+2b^2-22b+18bc-(1-b)\sqrt{4b^2+(1-9c)b+4})}{729b^2}$.

Figure 8 and Lemma A9 show the baseline results, which indicates that the impacts of network effects characterized in Corollary 1 of main body are robust. First, as the network effect increases, the profits of value chain participants become higher. Second, strengthening of network effects reduces the efficiency loss caused by double marginalization.

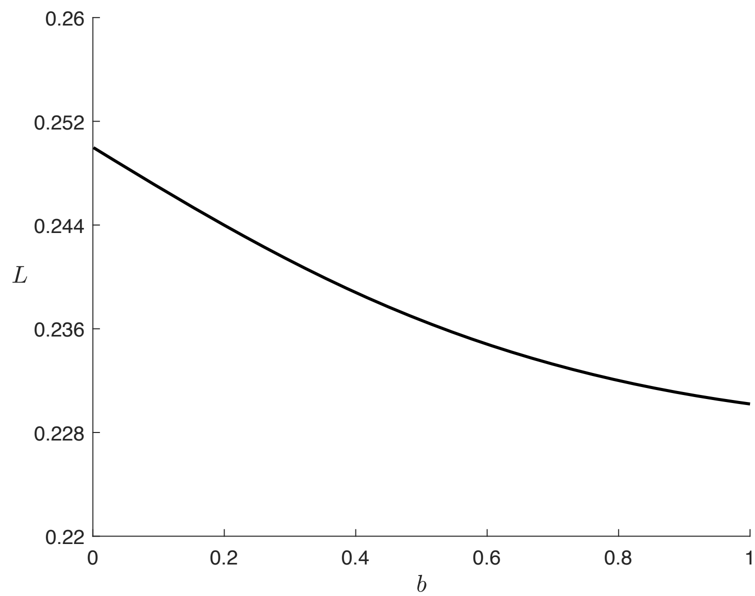

Figure 8: The Profits and Efficiency Loss of the Value Chain

E.2 Expanding the Network via Seeding

Now, we investigate the case of expanding networks through seeding strategies. The profit of central planner becomes $\Pi_I := [\theta(1 + b(1 - \hat{\theta} + \alpha)) - c](1 - \hat{\theta}) - c\alpha$. In the decentralized, the retailer's and manufacturer's profits are $\Pi_R(\alpha_R, \hat{\theta}; \alpha_M, w) := [\theta(1+b(1-\hat{\theta}+\alpha_R+\alpha_M))-w](1 - \hat{\theta}) - w\alpha_R$ and $\Pi_M(\alpha_M, w) := (w - c)(1 - \hat{\theta} + \alpha_R) - c\alpha_M$, respectively, subject to $0 \le \alpha_R + \alpha_M \le \hat{\theta} \le 1$. Proposition A2 below characterizes the equilibrium of each case.

Proposition A2. *With seeding, there exists a unique equilibrium such that*

i. (centralized case) The central planner's optimal strategy satisfies that given $c^I := \frac{b}{4}$,

(i) When $c < c^I$, $\alpha_I = \frac{1}{2}$, $\hat{\theta}_I = \frac{1}{2}$, *thus,* $\Pi_I = \frac{1+b}{4} - c$;

(ii) When $c \geq c^I$, $\alpha_I = 0$, $\hat{\theta}_I = \frac{1+2b-\sqrt{b^2+(1-3c)b+1}}{3b}$, *thus,*
$\Pi_I = \frac{(b-1+\sqrt{b^2+(1-3c)b+1})(1+b^2+4b-6bc-(1-b)\sqrt{b^2+(1-3c)b+1})}{27b^2}$.

ii. (decentralized case) The equilibrium of the decentralized case satisfies that

(i) When $c < c^M$, $\alpha_M^* = \frac{2+4b-\sqrt{4b^2+7b+4}}{3b}$, $w^* = \frac{(2\sqrt{4b^2+7b+4}+b+2)(\sqrt{4b^2+7b+4}-b-2)}{9b}$, $\alpha_R^* = 0$, $\hat{\theta}^* = \frac{2+4b-\sqrt{4b^2+7b+4}}{3b}$, *thus,* $\Pi_M = \frac{(8b^2+14b+8)\sqrt{4b^2+7b+4}-11b^3-27b^2c-39b^2-42b-16}{27b^2}$,
$\Pi_R = \frac{(2+b-\sqrt{4b^2+7b+4})(\sqrt{4b^2+7b+4}-5b^2-8b-2)}{27b^2}$;

(ii) When $c \geq c^M$, $\alpha_M^* = 0$, $w^* = \frac{4+4b^2+10b+9bc-2(1-b)\sqrt{4b^2+(1-9c)b+4}}{27b}$, $\alpha_R^* = 0$, $\hat{\theta}^* = \frac{2+7b-\sqrt{4b^2+(1-9c)b+4}}{9b}$, *thus,* $\Pi_M = \frac{(4+4b^2+10b-18bc-2(1-b)\sqrt{4b^2+(1-9c)b+4})(\sqrt{4b^2+(1-9c)b+4}-2(1-b)}{243b^2}$,
$\Pi_R = \frac{(2-2b-\sqrt{4b^2+(1-9c)b+4})(2+2b^2-22b+18bc-(1-b)\sqrt{4b^2+(1-9c)b+4})}{729b^2}$,

where c^M *is the unique root of* $\frac{(8b^2+14b+8)\sqrt{4b^2+7b+4}-11b^3-27b^2c-39b^2-42b-16}{(4+4b^2+10b-18bc-2(1-b)\sqrt{4b^2+(1-9c)b+4})(\sqrt{4b^2+(1-9c)b+4}-2(1-b)} = \frac{1}{9}$.

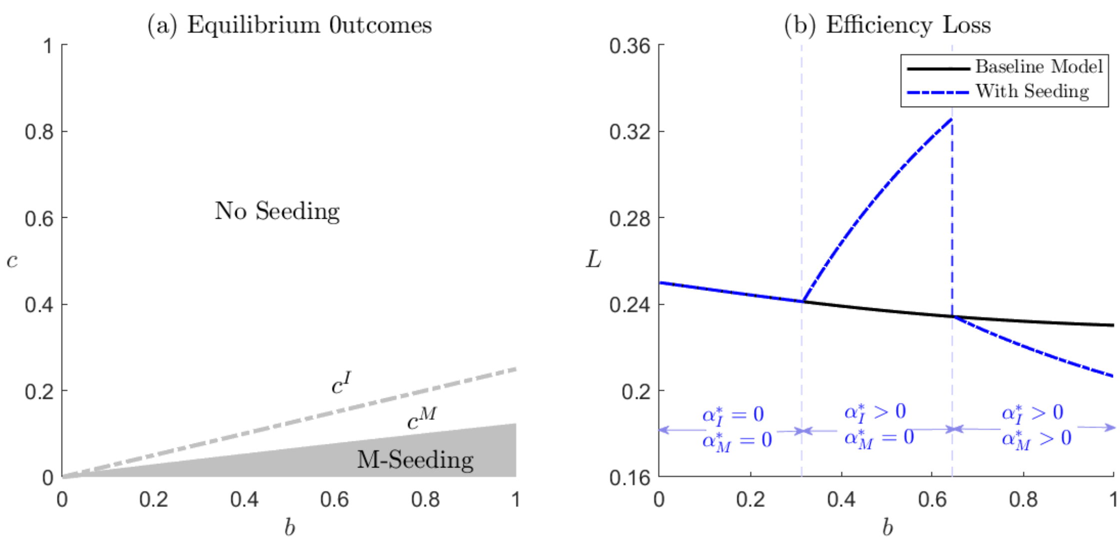

Figure 9: The Equilibrium Outcomes with Seeding and the Efficiency Loss

Figure 9(a) visualizes the equilibrium outcome given by proposition A2. We find that the conclusions given in Section 4.1 still hold when the network effects are heterogeneous. First, the value chain can still monetize the network effect by expanding the user base by seeding strategy. This occurs when marginal costs are sufficiently small. Second, it is never an equilibrium for the manufacturer and the retailer to seed the market simultaneously. In fact, only the manufacturer adopts seeding strategies.

Figure 9(b) shows the results of the efficiency loss compared to the baseline model. Similar to the findings revealed in Proposition 4, the impact of introducing seeding on the value chain's efficiency is also non-monotonic in the heterogeneous network effects case.

E.3 The Generalized Model

Next, we consider the generalized model which incorporates both seeding and engineering strategies to further extend the network effects. Due to the computational complexity, we are only able to obtain the results through numerical analysis.

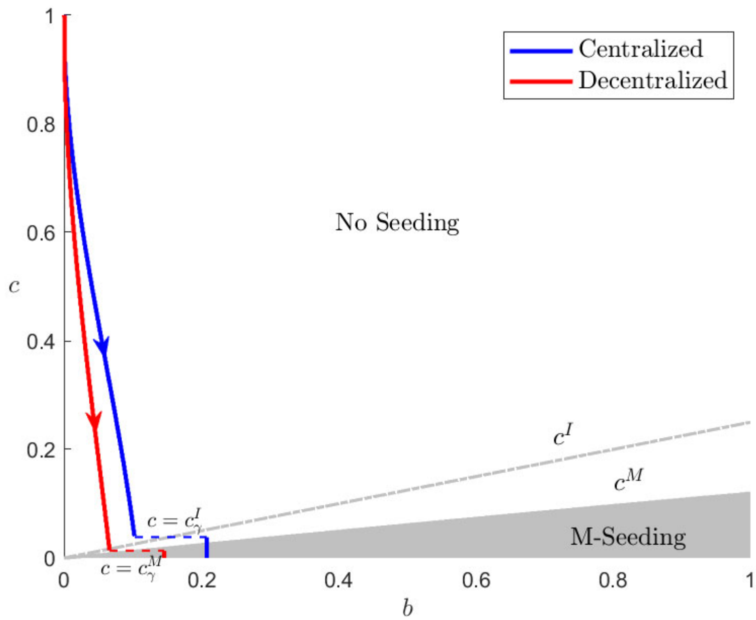

Figure 10: The Equilibrium Outcomes of the Generalized Model

As Figure 10 shows, the optimal strength of network effects does note change continuously. That is, there is a jump when the decision maker switches from no-seeding strategy to seeding strategy, which is consistent with the results of the main body. Note that as the optimal b increases horizontally towards the right side, it also falls into the seeding region both in the centralized and decentralized cases. Therefore, similar to Proposition 2, the complementary relationship between the two strategies is still present.