Relational Contracts in Usage-Based Insurance*

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Abstract

In light of the burgeoning usage-based insurance (UBI) market, we study relational contracts with moral hazard in a competitive insurance market. The insurer can use an objective and a subjective signal about the insured's behavior as explicit and implicit incentives, respectively. We show that with limited liability the subjective signal may be ignored even in a long-term contract when it is relatively imprecise. Moreover, the objective and subjective signals can be both complements and substitutes. Whereas a more precise subjective signal can always improve the insurance market efficiency, the welfare implication of the objective signal can be non-monotonic. In particular, when a more precise objective signal leads to a sufficiently efficient explicit contract, it may reduce the efficiency of the relational contract, or even render all relational contracts infeasible. Our results suggest that regulatory policies that enhance the enforceability of subjective signals can mitigate the distortion in the design of UBI contracts and in the ex-ante investment in related monitoring technologies.

Keywords: Relational Contract, Usage-based Insurance, Monitoring Technology

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1 Introduction

Insurance markets are plagued by moral hazard. A typical remedy is to monitor the insured individual's action so that the insurer can incentivize the insured using the collected data. In traditional insurance markets, objective signals, which could be grouped into ex-ante signals (e.g., the insured's demographic data) and ex-post signals (e.g., official accident reports), are commonly used in contract design, but have often limited precision in measuring the insured's action. Recently, the rise of the Internet of Things and big data technology is reshaping the way insurance companies monitor their policyholders. These technologies allow insurers to collect more precise, real-time, and granular data about the insured's action. For example, road monitoring systems can easily detect risky actions and facilitate accident investigations, meaning that objective signals have been increasingly precise. Furthermore, in-car telematics and drivers' smartphones are now used to monitor drivers' real-time driving behavior, which give rise to novel usage-based insurance (UBI) such as pay-how-you-drive plans. UBI plans have been increasingly popular as insurers recognize the benefits of using digital technologies to mitigate moral hazard. It is reported that insurance companies and smart car maker Tesla are investing heavily in UBI,[1] and that related products are expanding exponentially.[2]

In a typical UBI program, the insurer uses in-car telematics or the driver's smartphone app to monitor the insured's real-time driving behavior and record any risky actions, such as speeding, harsh braking, phone distraction and so forth. The insurer then calculates an individual driving score based on these data and adjusts the insured's premium accordingly. Typically, if the driving score falls below a certain threshold, the insured will face a premium increase as a penalty. These additional UBI signals thus seem to imply better monitoring, more efficient contracts, and higher welfare for all parties involved. Indeed, several empirical studies have shown that UBI improves the users' driving safety (Soleymanian et al., 2019; Reimers and Shiller, 2019; Jin and Vasserman, 2021).

In contrast to objective signals, UBI signals are often perceived as subjective measures of the driver's performance for the following reasons. First, some key measures, such as harsh braking and phone distraction, are difficult to evaluate objectively and may sometimes be

[1] Elon Musk, the CEO of Tesla, announced in a company earnings call in October 2020 that Tesla is building a major insurance company (`https://www.forbes.com/advisor/car-insurance/tesla-insurance/`). Tesla's insurance is based on real-time driving behavior and Safety Score, as disclosed on the company's website (`https://www.tesla.com/support/insurance/real-time-insurance`).

[2] According to the report by Vantage Market Research, "Usage-Based Insurance (UBI) Market – Global Industry Assessment & Forecast", the global UBI market was valued at USD 22.79 billion in 2022 and is projected to reach USD 150.45 billion by 2030, with a compound annual growth rate of 26.60%.

impacted by factors beyond the driver's control.[3] More important, how insurers calculate the insured's driving score and relate the score to the insured's premium is usually opaque, and may be intentionally withheld and manipulated by the insurer. Such concerns have sparked widespread disputes among UBI policyholders regarding driver evaluation and subsequent changes in premium.[4] Nevertheless, due to the relatively small amount of money involved, policyholders typically have little incentive to pursue legal action individually. In contrast, collective legal action has been taken, as evidenced by two ongoing class-action lawsuits filed against UBI companies in the U.S.—one in 2022 and another in 2023,[5] which allege that the insurers' scoring systems are unfair and that the premiums are inflated. Furthermore, some policyholders have even considered quitting the program after getting an inflated premium.[6] These facts indicate that UBI signals are substantially subjective; thus, an incentive scheme based on such signals will have much weaker enforceability than with objective signals.

Therefore, an efficient UBI contract relies on the mutual trust and expectation of future surplus from continued transaction between the insurer and the insured. In other words, a sustainable UBI contract must be a self-enforcing agreement between the contracting parties. This paper characterizes the optimal relational insurance contract employing objective and UBI (subjective) signals. Our primary goal is to answer the following questions. Under what conditions will such relational insurance contracts emerge? What is the optimal interplay between objective and subjective signals? How does such interplay depend on the precision of each signal? Finally, how will advances in digital technology impact the optimal relational contract and the insurance market efficiency?

In the model, a competitive insurance company (the principal) offers a relational contract to a risk-averse individual (the agent). In each period, the agent faces a possible financial loss l and can choose a precautionary effort level. The probability of loss depends on the agent's effort and a persistent external state. After the effort has been chosen, the principal receives

[3] For example, the driver-assistance system will sometimes automatically slow or stop the vehicle based on inaccurate prediction of the probability of collision.

[4] Users' reviews of UBI apps in the Apple Store show prevalent complaints about driving performance measures and premium changes in UBI programs, including American Family Insurance's Knowyourdrive, State Farm's Drive Safe and Saving, Farmers Insurance's Signal, UCCA's SafePilot, and Progressive Snapshot.

[5] The 2022 lawsuit was Shawn Schneider vs. State National Insurance Company, and the 2023 lawsuit was Joshua Santiago vs. Tesla. In both cases, the plaintiffs complain about the inflated premiums based on low Safety Score due to "ghost" forward collision warnings, as well as other inaccurate assessments.

[6] For example, an user of Knowyourdrive commented after receiving a 50% discount cut, "According to the app, I'm driving safely, but I'm not seeing much of a financial benefit. [...] Is it worth it?" (`https://apps.apple.com/us/app/knowyourdrive/id1235726818`).

a binary UBI signal about the agent's effort; the probability of a good signal increases in the agent's effort. In addition, if a loss occurs, the principal also receives an objective signal about whether the agent is responsible for the loss, which may be a false negative or a false positive with probability $1-p$ and $1-q$, respectively, where $p, q \in (0.5, 1)$. The principal pays the agent an indemnity κ_ι if the signal indicates that the agent is responsible, and an indemnity κ_ε otherwise. In the absence of loss, the agent is prescribed to pay a penalty β if the principal receives a bad UBI signal. If the agent reneges on the penalty, the two parties permanently switch to a spot contract employing only the objective signal.[7] We characterize the stationary relational contract that maximizes the agent's payoff (social welfare).

As a benchmark and the fallback of relational contract, the optimal spot contract balances risk sharing and incentive provision. Under the monotone likelihood ratio property (MLRP) of the objective signal, this trade-off manifests as positive but partial coverage in the optimal contract: $0 \leq \kappa_\iota < \kappa_\varepsilon < l$. Since the signal that external conditions caused the loss is better news for the agent's effort, the indemnity κ_ε is larger than κ_ι. Whereas the indemnity κ_ε is always positive, κ_ι may bind at zero when the incentive effect is significant. In this case, the principal would have set a negative κ_ι to leverage financial incentive without the constraint of limited liability. Instead, she sacrifices risk sharing for more incentive provision by raising the premium, thereby raising the agent's marginal utility of money. As the objective signal becomes more precise (i.e., as p or q increases), the agent's payoff (social welfare) from the optimal spot contract increases monotonically, since moral hazard is better mitigated.

We now turn to the optimal relational contract. When the agent is sufficiently patient, the self-enforcing constraint is slack, and the resultant efficient relational contract is equivalent to the optimal long-term contract employing both the objective and UBI signals. Notably, when the UBI signal is relatively noisy and the optimal spot contract exhibits $\kappa_\iota = 0$, the UBI signal may not be employed; that is, the penalty β is zero even if it were enforceable. This may seem to contradict the classic informativeness principle (Holmstrom, 1979), which suggests that the UBI signal should be used despite its noisiness since it provides additional information about effort. However, this insight relies on the optimal contract being interior. When $\kappa_\iota = 0$ in the optimal spot contract, as argued above, the principle raises the premium thereby harming risk sharing, and a positive β will aggravate this issue. If the UBI signal is relatively imprecise, the negative effect on risk sharing may outweigh the value of additional information, and thus, the signal is ignored. As the objective signal becomes more precise,

[7] In many insurance companies, a policyholder is automatically enrolled in the conventional auto insurance plan after quitting the UBI program.

incentives can be more efficiently provided, allowing the principal to charge a lower premium. As a result, the implicit incentive β can be used to enhance incentives, suggesting that the objective and UBI signals are complementary. In contrast, the two signals can be substitutes when the UBI signal is precise. In this case, the implicit incentive is relatively effective, and is substituted by the explicit incentive as the objective signal becomes more precise.

Furthermore, when the UBI and objective signals become more precise, they may impose qualitatively different impacts on the relational contract and the insurance market efficiency. First, social welfare is monotonically increasing in the precision of the UBI signal, irrespective of whether the self-enforcing constraint is binding. This is because a more precise UBI signal leads to a more efficient implicit incentive, yielding greater additional surplus compared to the spot contract. This can relax a binding self-enforcing constraint, which further enhances the contract efficiency by reducing the distortion in the combined use of explicit and implicit incentives. By contrast, the welfare implication of the objective signal can be non-monotonic. On the one hand, a more precise objective signal leads to a more efficient relational contract provided that the contract is self-enforcing, since the principal can use the objective signal to provide more effective incentives to the agent. On the other hand, a more precise objective signal can also increase the return of the fallback, worsening the self-enforcing constraint. If the self-enforcing constraint is already tightly binding, then the negative effect of the more precise objective signal may outweigh the positive effect, rendering a relational contract less efficient, or even no longer self-enforcing.

Intuitively, when the spot contract becomes relatively attractive, the principal needs to reduce the agent's reneging temptation to maintain self-enforcing. One plausible solution is to lower the premium, thereby rendering the risk-averse agent less sensitive to the penalty. However, this would also dampen the incentives across all states. It turns out that a more effective way is to cut the penalty. In particular, any contract with positive penalty cannot be self-enforcing if the fallback is too attractive. As such, the self-enforcing constraint leads to either a less efficient relational contract (an intensive-margin distortion) or non-existence of relational contract (an extensive-margin distortion).

Our paper thus has meaningful implications for the insurance market in the digital era. From the positive perspective, our paper characterizes the conditions under which the novel UBI can emerge. This allows us to explain why UBI's market penetration varies significantly across economies. In particular, economies equipped with more advanced traffic information infrastructures, e.g., extensive road monitoring systems and pervasive speed cameras, may be less likely to foster a thriving UBI market. This may explain why UBI is still in the early stage in countries such as China (e.g., see Zhuo and Huang 2019 and Wang et al. 2023).

From the normative perspective, our paper sheds light on the benefits of regulating novel insurance policies such as UBI. The welfare superiority of the optimal long-term contract suggests that it is socially beneficial to strengthen the enforceability of UBI contracts, for example by improving the transparency of the scoring algorithm and pricing terms of UBI. That is, a more efficient outcome can be achieved by mitigating the distortion in the contract design. In the long run, regulations may further mitigate the investment distortion of both the UBI and objective signals. Suppose before the insurer offers a contract, a social planner can invest in the precision of both signals. Under weak enforceability of UBI contracts, there may be underinvestment in the objective signal because of the concerns about weakening the contract's efficiency or feasibility due to the self-enforcing constraint. In contrast, both underinvestment and overinvestment may occur for the UBI signal. On the one hand, if no self-enforcing relational contract exists due to relatively precise objective signals, then there may be no investment in UBI signal at all. On the other hand, if the self-enforcing constraint binds, then the marginal benefit of investment will be higher than the first-best level due to the additional benefit of relaxing the constraint, leading to overinvestment. In practice, it is common for the government to invest in the objective signals, while the firms invest in the subjective signals. The above argument suggests that public investment in such monitoring technologies may either crowd out or excessively stimulate private investment. Thus, polices that enhance the enforceability of these subjective performance measures can mitigate the distortion in monitoring technology investment, as well as that in contract design.

1.1 Related Literature

Our paper builds on a rich literature on insurance under ex-ante moral hazard, initiated by the seminal works of Arrow (1978), Shavell (1979a,b), Holmstrom (1979), among others.[8] The general insight is that the optimal contract balances the risk-sharing benefits of more insurance and the incentive benefits of less insurance, which typically leaves the policyholder partially insured. Existing studies usually focus on formal insurance contracts. In contrast, we consider relational insurance contracts, and examine how the signals about the insured's effort that have different legal enforceability and advances in monitoring technology affect the optimal insurance contracts, which is understudied in the existing insurance literature. Different from our perspective, Bourgeon and Picard (2020) employ the incomplete contract framework to examine how insurance law can mitigate moral hazard.

[8] Winter (2000) provides an excellent survey of the theoretical works on insurance under moral hazard.

Our paper also contributes to the large literature on relational contracts (e.g., Bull 1987; MacLeod and Malcomson 1998; Baker et al. 2002; Levin 2003; Rayo 2007); see Malcomson (2013) and MacLeod (2022, Chapter 9) for reviews. Our paper is most closely related to the branch of this literature that studies the interaction between explicit and implicit contracts with moral hazard (e.g., Baker et al. 1994; Schmidt and Schnitzer 1995; Che and Yoo 2001).[9] Baker et al. (1994) show that explicit contracts can improve the efficiency of continued trade, but can also crowed out relational contracts by increasing the attractiveness of the fallbacks. In the context of UBI, we show analogously that the objective and subjective signals of the insured's behavior can be both complements and substitutes. In particular, a more precise objective signal can distort relational contracts on both the intensive and extensive margins. However, in contrast to the risk-neutral settings of those papers, we consider a risk-averse agent, and thus, the wealth effect plays a substantial role in balancing risk sharing, incentive provision and self-enforcement. Pearce and Stacchetti (1998) and Thomas and Worrall (1994, 2018) consider risk-averse agents whose actions are observable. In contrast, we consider risk aversion with moral hazard and examine the impacts of the objective and subjective signals on mitigating moral hazard and the efficiency of relational contracts.

Our paper is also related to the literature that studies how performance measures affect the effectiveness of implicit incentives. Meyer and Vickers (1997) study career concerns with a ratchet effect, and show that comparative performance information can worsen the ratchet problem and outweigh the insurance gain in certain information structures. Deb et al. (2016) show that whereas private peer evaluations can enhance the efficiency of relational contracts, they should be used sparingly because truthful revelation of information necessitates surplus destruction. Fong and Li (2017) show that intertemporal garbling of performance measures can improve the trade-off between motivating the agent and easing the principal's reneging temptation. Similarly, we show that less information can be sometimes beneficial. That is, the UBI signal may be ignored even in a long-term contract since the negative effect on risk sharing may dominate the value of additional information, and a better objective signal may weaken the efficiency of relational contract by worsening the self-enforcing constraint.

The rest of the paper is organized as follows. After setting up the model in Section 2, we characterize in Section 3 the optimal spot contract, i.e., the fallback of relational contract. In Section 4, we characterize the optimal relational contract, and investigate how the objective and UBI signals affect the contract and social welfare. Section 5 discusses policy implications. Finally, Section 6 concludes the paper. All proofs are provided in the appendix.

[9] Corts (2018) provides an excellent review of interaction between explicit and implicit contracts.

2 Model

Players and actions. Consider a competitive insurance market. A representative insurance company (*principal*) offers coverage to a risk-averse individual (*agent*). Time is discrete and infinite, $t \in \{1, 2, \dots\}$. Both parties share a discount factor $\delta \in (0, 1)$. In each period, the agent faces a possible financial loss, which is given by a constant $l > 0$. Let $\omega_t \in \{L, N\}$ be the outcome of period t, where L means that there is a loss and N means no loss. The probability of loss in period t is jointly determined by the agent's precautionary effort e_t in t and an external state θ, for example the agent's age, car condition, address, and so forth. We assume that $e_t \in [0, 1]$ and $\theta \in (0, 1)$, and that θ is constant across time. Specifically, a loss may result from either an internal cause (e.g., the agent's risky driving behavior) or an external cause (e.g., poor traffic condition), or both, which occurs with probability

$$Pr(\omega_t = L) = \underbrace{(1 - e_t)\theta}_{\text{by internal cause}} + \underbrace{e_t(1 - \theta)}_{\text{by external cause}} + \underbrace{(1 - e_t)(1 - \theta)}_{\text{by both causes}} = 1 - e_t\theta.$$

Thus, the agent incurs no loss in the absence of both causes, with probability $e_t\theta$. In each period, the principal offers an insurance policy to the agent, which will be formally defined later. The agent chooses whether to accept the offer. If the agent accepts the offer, then he files a claim in that period whenever there is a loss.

Information. The external state θ is publicly observed at the beginning of the game, while the agent knows his effort level privately. If the agent accepts the offer and a loss occurs, the principal will receive an objective signal x_t (e.g., an official accident report) about the cause of the loss. We assume that $x_t \in \{\iota, \varepsilon\}$, where signal ι means that the loss results fully or partially from an internal cause, and signal ε means that it results fully from an external cause. The objective signal x_t is informative and imperfect in the sense that

$$Pr(x_t = \iota | \text{the loss is truly related to an internal cause}) = p,$$
$$Pr(x_t = \varepsilon | \text{the loss is truly unrelated to an internal cause}) = q,$$

with $p, q \in (0.5, 1)$. Thus, the probability of false positive for an internal cause is $1 - q$, and that of false negative is $1 - p$. Clearly, the precision of x_t is increasing in both p and q. Then, the interim probabilities of receiving signals ι and ε are given by, respectively,

$$\phi_\iota := (1 - e_t)[\theta + (1 - \theta)]p + e_t(1 - \theta)(1 - q),$$
$$\phi_\varepsilon := e_t(1 - \theta)q + (1 - e_t)[\theta + (1 - \theta)](1 - p).$$

Let $\phi_n := 1 - \phi_\iota - \phi_\varepsilon = e_t\theta$ be the interim probability of no loss.

Payoffs. The agent has a constant initial wealth $w > l$ in each period, and has no access to credit. His utility over wealth and effort is given by $u(w) - h(e)$, with $u' > 0$, $u'' < 0$, $h' > 0$ if $e > 0$, $h'' > 0$, and $h(0) = h'(0) = 0$. The principal has no costs other than the coverage.

Spot contract. The principal can always offer the agent a spot contract, which specifies a premium $r(\theta) \geq 0$ and a level of coverage $k(\theta, x_t) \geq 0$. This reflects that both the external state and the objective signal are contractible, whereas the agent's effort is not. To simplify notation, suppress θ in the contract, and let k_ι and k_ε be the coverage upon the signals ι and ε, respectively. An optimal spot contract solves the following program.

$$\max_{r,k_\iota,k_\varepsilon,e} U^s := \phi_n u(w-r) + \phi_\iota u(w-r-l+k_\iota) + \phi_\varepsilon u(w-r-l+k_\varepsilon) - h(e)$$

subject to the agent's incentive compatibility constraint:

$$e \in \operatorname*{argmax}_{\tilde{e}} \tilde{\phi}_n u(w-r) + \tilde{\phi}_\iota u(w-r-l+k_\iota) + \tilde{\phi}_\varepsilon u(w-r-l+k_\varepsilon) - h(\tilde{e}), \tag{IC-A}$$

where each $\tilde{\phi}$ is calculated at $\tilde{e}$, and the principal's nonnegative expected profit constraint:

$$\Pi^s := r - \phi_\iota k_\iota - \phi_\varepsilon k_\varepsilon \geq 0. \tag{IR-P}$$

Note that the agent's participation constraint is automatically satisfied since the principal can guarantee him the utility of self-insurance by offering zero coverage for zero premium. Let $\{r^*, k_\iota^*, k_\varepsilon^*, e^s\}$ be an optimal spot contract, and V^s be the agent's associated utility.

First-best benchmark. Suppose the agent's effort were observable and contractible, then it is well known that an optimal contract provides full-insurance to the agent: $k_\iota^{fb} = k_\varepsilon^{fb} = l$. It follows that an optimal contract $\{r^{fb}, e^{fb}\}$ solves:

$$\max_{r,e} u(w-r) - h(e)$$

subject to $r \geq (1-\phi_n)l$. Evidently, at the optimum the principal charges an actuarially fair premium, i.e., $r = (1-e\theta)l$. Thus, the first order condition of e is given by

$$u'(w-r)\phi_n' l = h'(e), \tag{1}$$

where $\phi_n' := \partial\phi_n/\partial e$. That is, the marginal utility from premium reduction due to an higher effort level must equal the marginal cost of effort. Applying the implicit function theorem to (1) with respect to e and l, we have

$$\frac{de^{fb}}{dl} = \frac{[u' - (1-e^{fb}\theta)lu'']\theta}{h'' - (\theta l)^2 u''} \geq 0.$$

Thus, the first-best effort is nondecreasing in the magnitude of loss. Intuitively, the greater the loss is, the more premium can be reduced by an extra unit of effort. Furthermore, given the effort level, the greater the loss, the lower the net wealth $w - r$, and thus, the higher marginal utility. Therefore, the marginal benefit of effort is nondecreasing in the loss.

Then, consider the relationship between e^{fb} and θ. Similarly, we have

$$\frac{de^{fb}}{d\theta} = \frac{(u' + e^{fb}\theta l u'')l}{h'' - (\theta l)^2 u''} \geq 0 \Longleftrightarrow -\frac{u''}{u'} \leq \frac{1}{e^{fb}\theta l}.$$

Thus, the first-best effort is nondecreasing in the external state if and only if the degree of absolute risk aversion is sufficiently low. On the one hand, the better the external state, the more effective the agent's effort in reducing the expected loss. On the other hand, given the effort, the better the external state, the higher the net wealth $w - r$ and the lower marginal utility. Thus, whether a better external state leads to a higher effort level depends on which of the two countervailing effects dominates the other.

In respect of welfare, it follows immediately from the envelope theorem that the agent's utility (social welfare) is decreasing in the loss, and increasing in the external state.

3 The Optimal Spot Contract

In this section, we consider spot contracts. Note that the agent's problem is strictly concave. As a result, the first order condition is necessary and sufficient for the incentive constraint. Similarly, define $\phi'_\iota := \partial\phi_\iota/\partial e$ and $\phi'_\varepsilon := \partial\phi_\varepsilon/\partial e$. To simplify notation, let $u_n = u(w - r)$, $u_\iota = u(w - r - l + k_\iota)$ and $u_\varepsilon = u(w - r - l + k_\varepsilon)$. The Lagrangian is given by

$$\begin{aligned}\mathcal{L} = &\ \phi_n u_n + \phi_\iota u_\iota + \phi_\varepsilon u_\varepsilon - h(e) + \lambda(r - \phi_\iota k_\iota - \phi_\varepsilon k_\varepsilon) \\ &+ \mu\left[\phi'_n u_n + \phi'_\iota u_\iota + \phi'_\varepsilon u_\varepsilon - h'(e)\right] + \nu_r r + \nu_\iota k_\iota + \nu_\varepsilon k_\varepsilon,\end{aligned}$$

where λ and μ are the respective Lagrangian multipliers for the IR-P and IC-A constraints, and $\nu_r, \nu_\iota, \nu_\varepsilon$ are the multipliers for the nonnegativity constraints of $r, k_\iota, k_\varepsilon$, respectively.

Deriving the first order conditions for the controls and rearranging, we have

$$u'_n = \left(\lambda + \frac{\nu_r + \nu_\iota + \nu_\varepsilon}{\phi_n}\right) \Big/ \left(1 + \mu\frac{\phi'_n}{\phi_n}\right), \tag{2}$$

$$u'_\iota = \left(\lambda - \frac{\nu_\iota}{\phi_\iota}\right) \Big/ \left(1 + \mu\frac{\phi'_\iota}{\phi_\iota}\right), \tag{3}$$

$$u'_\varepsilon = \left(\lambda - \frac{\nu_\varepsilon}{\phi_\varepsilon}\right) \Big/ \left(1 + \mu\frac{\phi'_\varepsilon}{\phi_\varepsilon}\right). \tag{4}$$

Before further analysis, we first lay out two useful lemmas. The first lemma shows that the objective signal x_t satisfies the monotone likelihood ratio property (MLRP).

Lemma 1. *The objective signal x_t satisfies MLRP: for any $p, q \in (0.5, 1)$ and $e, \theta \in (0, 1)$,*

$$\frac{\phi'_\iota}{\phi_\iota} < \frac{\phi'_\varepsilon}{\phi_\varepsilon} < \frac{\phi'_n}{\phi_n}, \text{ with } \frac{\phi'_\iota}{\phi_\iota} < 0 < \frac{\phi'_n}{\phi_n}.$$

In particular, $\frac{\phi'_\varepsilon}{\phi_\varepsilon} \to \frac{\phi'_n}{\phi_n}$ as $p \to 1$, and $\frac{\phi'_\iota}{\phi_\iota} \to \frac{\phi'_\varepsilon}{\phi_\varepsilon}$ as $p, q \to 0.5$.

As is well known, MLRP means that the realization of a high signal (in ascending order as $x_t = \iota$, $x_t = \varepsilon$ and $\omega_t = N$) indicates that the agent's effort is more likely to be high. In particular, if $p = 1$, then an internal cause of loss will always be detected; thus, $x_t = \varepsilon$ is as informative as $\omega_t = N$ about the agent's effort. If $p = q = 0.5$, then the principal knows only whether there is an accident, but cannot infer the agent's effort from the signal.

Given that MLRP holds, we have the following lemma.

Lemma 2. *The Lagrangian multipliers $\lambda, \mu > 0$ and $\nu_r = \nu_\varepsilon = 0$.*

Then, the proposition below characterizes the optimal spot contract.

Proposition 1. *The optimal spot contract satisfies (2) to (4) with $\nu_r = \nu_\varepsilon = 0$, such that $r^* > 0$, $0 \le k^*_\iota < k^*_\varepsilon < l$, and $e^s > 0$. In particular, if $\phi'_\varepsilon \ge 0$, i.e., $(1-\theta)q \ge 1-p$, then $k^*_\iota > 0$. Furthermore, as $p, q \to 0.5$, $0 < k^*_\iota = k^*_\varepsilon < l$ in the limit.*

Proposition 1 states that under moral hazard full insurance is always suboptimal; on the other hand, positive coverage is desired even in the presence of moral hazard. The intuition is well known; that is, the optimal spot contract balances risk sharing and incentive providing. Moreover, under MLRP the coverage is increasing in the signal, since a higher signal implies a higher probability that the agent chooses a high effort. Whereas the agent always receives a positive coverage upon the signal ε, he may receive zero coverage upon ι as the maximal punishment for a low signal when the incentive effect is relatively strong. Figure 1 illustrates examples of k^*_ι and k^*_ε for p varying from 0.5 to 1. As depicted, k^*_ι is uniformly below k^*_ε, and is fixed at 0 in some region of p. Note that when $\phi'_\varepsilon \ge 0$, i.e., a higher effort level leads to a higher chance of paying k_ε, we have $k^*_\iota > 0$ unambiguously. This is because if $k_\iota = 0$, then a higher effort will tighten both the nonnegative-profit and incentive compatibility constraints, leading to zero effort, which is suboptimal. Intuitively, $\phi'_\varepsilon \ge 0$ means that p, q are relatively large, i.e., the objective signal is relatively informative. This enables the principal to better monitor the agent thereby in turn better insuring the agent. Note too that when $p = q = 0.5$,

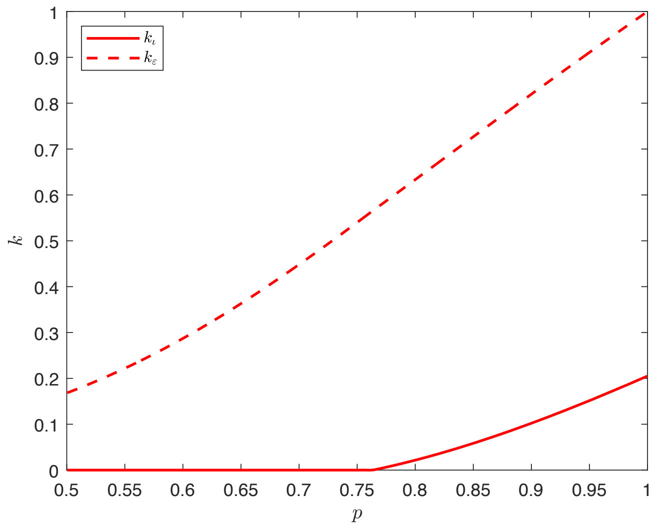

Figure 1: **The Optimal Spot Contract.** This figure illustrates the coverage of the optimal spot contract for $p \in [0.5, 1]$. The figure assumes that $u(w) = \log(w)$, $h(e) = e^2/10$, $w = 5$, $l = 1$, $\theta = 0.75$, and $q = 0.6$.

the objective signal is uninformative; thus, ω_t is a sufficient statistic for (ω_t, x_t) so that x_t is ignored. Moreover, since the agent's action only affects the probability of loss, the optimal contract entails a deductible (Holmstrom, 1979, Proposition 2).

As comparative statics, we examine how the precision of objective signal and the external state affect the optimal spot contract. In the digital era, for example, as more traffic cameras are installed, investigation reports of car accidents have been more accurate, that is, p and q increase. Meanwhile, due to the spread of more advanced driver-assistance systems, drivers can better prevent accidents, meaning that the external state θ raises. Then, how will such technological advancements affect the insurance contract? While it is difficult to derive sharp general results, we present some limit properties of the coverage as the signal precision and the external state approach perfection in the corollary below.

Corollary 1. *As the precision of the objective signal x_t and the external state θ approach perfection, the optimal spot contract satisfies that*

(i) $0 < \lim_{p \to 1} k_\iota^* < \lim_{p \to 1} k_\varepsilon^* = l$ *for any* $q \in (0.5, 1)$.

(ii) $0 \leq \lim_{q \to 1} k_\iota^* < \lim_{q \to 1} k_\varepsilon^* < l$ *for any* $p \in (0.5, 1)$.

(iii) $0 < \lim_{p,q \to 1} k_\iota^* < \lim_{p,q \to 1} k_\varepsilon^* = l$.

(iv) $0 < \lim_{\theta \to 1} k_\iota^* = \lim_{\theta \to 1} k_\varepsilon^* < l$, *and* $k_\varepsilon^* - k_\iota^*$ *decreases and converges to* 0 *as* $\theta \to 1$.

Corollary 1 states that as the probability of false negative of internal cause vanishes, i.e., as $p \to 1$, the optimal contract eventually covers fully the agent's loss upon the signal ε, but partially compensates him upon the signal ι. Intuitively, as $p \to 1$, the signal can perfectly reveal the agent's risky action. Thus, the most effective way for incentive providing and risk sharing is to offer the agent partial coverage when $x_t = \iota$, while full coverage when $x_t = \varepsilon$, since the latter case means that the loss is certainly due to an external cause. In contrast, if only the probability of false positive vanishes, that is, if only $q \to 1$, then it is optimal to provide partial coverage irrespective of the signal realization. Intuitively, with the possibility of false negative, i.e., $p < 1$, the optimal contract should punish the agent for both signals, offering higher coverage for a higher signal given that MLRP holds.

Similarly, if the external state were perfect, then a loss would only result from an internal cause, meaning that the outcome ω_t is sufficient for (ω_t, x_t). Thus, the objective signal should be ignored, and the optimal contract entails a deductible. This implies that as the external state approaches perfection, the difference in coverage eventually decreases.

Another relevant question is, how will the technological advancement affect the insured's payoff from the spot contract and social welfare? It follows from Lemma 1 that (IR-P) binds, and thus, $\Pi^s = 0$, meaning that V^s coincides with social welfare. The following proposition states that the insured will be better off and social welfare will be higher when the objective signal gets more precise and when the external state improves.

Proposition 2. *The agent's payoff (social welfare) from the spot contract, V^s, is increasing in both p and q, and is also increasing in the external state θ.*

Intuitively, as the objective signal becomes more precise, the moral hazard problem will be mitigated, thereby allowing the principal to better insure the agent. In addition, as the external state improves, for example because of better traffic conditions or more advanced driver-assistance systems, the agent is more capable of preventing losses, and thus, it is less costly to insure him. Consequently, the agent is better off and social welfare is higher.

4 Subjective Signal and Relational Contracts

In this section, we consider when both the principal and the agent can additionally observe a subjective signal y_t about the agent's effort in each period, indicating whether the agent had any risky actions, such as speeding, aggressive driving, following too closely and so on. Let $y_t = b$ denote the case when the subjective signal shows that the agent performed some risky action(s); $y_t = g$ otherwise. We assume that given the agent's effort e_t, the probability of a good signal is given by

$$Pr(g|e_t) = \psi(e_t) \in (0, 1),$$

with $\psi' > 0$ for any $e \in [0, 1]$; thus, a good signal is more likely under a high effort level.

As subjective, y_t is not contractible, i.e., no formal contract can be written based on y_t. Instead, the principal can offer a relational contract to the agent. For ease of exposition, we assume that if there is no loss in period t, the two parties can choose whether to adjust the current period's premium based on the realization of y_t: if $y_t = b$, the agent is prescribed to pay a penalty $\beta \geq 0$ at the end of t; otherwise, there will be no payment adjustments. Let $\phi_g := \phi_n\psi(e_t)$ and $\phi_b := \phi_n[1-\psi(e_t)]$ be the probability of no payment adjustments and that of the agent paying the penalty if he honors the agreement, respectively. To align with the practice, we focus on stationary relational contracts such that the amount of coverage and whether the premium will be adjusted depend only the realizations of the current period's signals (e.g, the UBI score in the current review period).

Thus, a relational contract specifies a premium $\rho(\theta) \geq 0$, a level of coverage $\kappa(\theta, x_t) \geq 0$, and a penalty $\beta \geq 0$. To simplify notation, suppress θ and x_t in the contract, and let κ_ι and κ_ε be the respective coverage for the signals ι and ε. Since we focus on stationary contracts, ρ, β, κ_ι and κ_ε are constant across time. Let $\{\rho^*, \kappa_\iota^*, \kappa_\varepsilon^*, \beta^*, e^r\}$ denote an optimal relational contract, which solves the following program.

$$\max_{\rho,\kappa_\iota,\kappa_\varepsilon,\beta,e} U^r := \phi_g u(w-\rho) + \phi_b u(w-\rho-\beta) + \phi_\iota u(w-\rho-l+\kappa_\iota) + \phi_\varepsilon u(w-\rho-l+\kappa_\varepsilon) - h(e)$$

subject to the agent's incentive compatibility constraint:

$$e \in \underset{\tilde{e}}{\operatorname{argmax}}\ \tilde{\phi}_g u(w-\rho) + \tilde{\phi}_b u(w-\rho-\beta) + \tilde{\phi}_\iota u(w-\rho-l+\kappa_\iota) + \tilde{\phi}_\varepsilon u(w-\rho-l+\kappa_\varepsilon) - h(\tilde{e}), \tag{IC-A}$$

the nonnegative expected profit constraint:

$$\Pi^r := \rho + \phi_b\beta - \phi_\iota\kappa_\iota - \phi_\varepsilon\kappa_\varepsilon \geq 0, \tag{IR-P}$$

and in addition the agent's dynamic enforcement constraint:

$$u(w-\rho) - u(w-\rho-\beta) \leq \frac{\delta}{1-\delta}(U^r - V^s). \tag{DE-A}$$

The (DE-A) constraint means that an optimal relational contract should be *self-enforcing*. In particular, if the agent reneges on the payment adjustment, then the two parties return to the optimal spot contract thereafter. Since the agent is risk averse, his reneging temptation depends on not only the endogenous variables ρ and β, but also the exogenous variable w and his preference. We assume that the agent's problem is strictly concave.[10]

Similarly, let $u_g = u(w-\rho)$ and $u_b = u(w-\rho-\beta)$, and with a slight abuse of notation, let $u_\iota = u(w-\rho-l+\kappa_\iota)$ and $u_\varepsilon = u(w-\rho-l+\kappa_\varepsilon)$. Note that if an optimal relational contract exhibits $\beta = 0$, then it coincides with the optimal spot contract. Hence, we ignore the nonnegativity constraint of β for the moment. Then, the Lagrangian is given by

$$\begin{aligned}\mathcal{L} = U^r &+ \lambda(\rho + \phi_b\beta - \phi_\iota\kappa_\iota - \phi_\varepsilon\kappa_\varepsilon) + \mu[\phi_g' u_g + \phi_b' u_b + \phi_\iota' u_\iota + \phi_\varepsilon' u_\varepsilon - h'(e)] \\ &+ \gamma\left[\frac{\delta}{1-\delta}(U^r - V^s) - u_g + u_b\right] + \nu_\rho\rho + \nu_\iota\kappa_\iota + \nu_\varepsilon\kappa_\varepsilon,\end{aligned}$$

where λ, μ and γ are the respective Lagrangian multipliers for (IR-P), (IC-A) and (DE-A), and ν_ρ, ν_ι and ν_ε are those for the nonnegativity constraints of ρ, κ_ι and κ_ε, respectively. Then, deriving the first order conditions and rearranging, we have

$$u_g' = \left(\lambda + \frac{\nu_\rho + \nu_\iota + \nu_\varepsilon}{\phi_g}\right) / \left(1 + \frac{\gamma\delta}{1-\delta} + \mu\frac{\phi_g'}{\phi_g} - \frac{\gamma}{\phi_g}\right), \tag{5}$$

$$u_b' = \lambda / \left(1 + \frac{\gamma\delta}{1-\delta} + \mu\frac{\phi_b'}{\phi_b} + \frac{\gamma}{\phi_b}\right), \tag{6}$$

$$u_\iota' = \left(\lambda - \frac{\nu_\iota}{\phi_\iota}\right) / \left(1 + \frac{\gamma\delta}{1-\delta} + \mu\frac{\phi_\iota'}{\phi_\iota}\right), \tag{7}$$

$$u_\varepsilon' = \left(\lambda - \frac{\nu_\varepsilon}{\phi_\varepsilon}\right) / \left(1 + \frac{\gamma\delta}{1-\delta} + \mu\frac{\phi_\varepsilon'}{\phi_\varepsilon}\right). \tag{8}$$

Similar to Section 3, we first layout two useful results. The following lemma shows that the subjective signal also satisfies MLRP. Formally,

Lemma 3. *The subjective signal y_t satisfies MLRP: for any $e \in [0,1]$,*

$$\frac{\phi_b'}{\phi_b} = \frac{\phi_n'}{\phi_n} - \frac{\psi'(e)}{1-\psi(e)} < \frac{\phi_n'}{\phi_n} < \frac{\phi_n'}{\phi_n} + \frac{\psi'(e)}{\psi(e)} = \frac{\phi_g'}{\phi_g}.$$

Given that y_t satisfies MLRP, we can measure the informativeness of y_t by the difference between $\frac{\phi_g'}{\phi_g}$ and $\frac{\phi_b'}{\phi_b}$. Note that for any $e_H > e_L$ with e_H close to e_L, we have

$$\log\left(\frac{\phi_g(e_H)}{\phi_g(e_L)} / \frac{\phi_b(e_H)}{\phi_b(e_L)}\right) \approx \left|\frac{\phi_g'(e_L)}{\phi_g(e_L)} - \frac{\phi_b'(e_L)}{\phi_b(e_L)}\right| (e_H - e_L).$$

[10] A sufficient condition for the concavity is that $(2\phi_n'\psi' + \phi_n\psi'')u(w) - h'' < 0$ for all $e \in [0,1]$.

It follows that the ratio of the likelihood ratio is increasing in $\left|\frac{\phi_g'}{\phi_g} - \frac{\phi_b'}{\phi_b}\right|$. That is, the measure is consistent with Blackwell's notion (Blackwell, 1951). We say that y_t is *more informative* with one monitoring technology than with another if the former has a greater value of the above measure than the latter for all $e \in [0,1]$. We will elaborate more on this point later.

Then, based on Lemma 3, we have the following similar result to Lemma 2.

Lemma 4. *The Lagrangian multipliers $\lambda, \mu > 0$ and $\nu_\varepsilon = 0$.*

As a reference point, suppose y_t is also contractible. In this case, the optimal long-term contract employing both x_t and y_t is the solution to the above Lagrangian with $\gamma = \nu_\varepsilon = 0$, provided that $\beta > 0$; if the solution exhibits $\beta \leq 0$, then the optimal long-term contract is identical to the optimal spot contract. Call such a contract the first-best relational contract, denoted by $\{\rho^{**}, \kappa_\iota^{**}, \kappa_\varepsilon^{**}, \beta^{**}, e^{**}\}$. The next proposition characterizes this contract.

Proposition 3. *The first-best relational contract satisfies (5) to (8) with $\gamma = \nu_\varepsilon = 0$, such that it takes either of the following forms:*

(i) $\beta^{**} = 0$, $\rho^{**} = r^* > 0$, $\kappa_\iota^{**} = k_\iota^* = 0$, $0 < \kappa_\varepsilon^{**} = k_\varepsilon^* < l$, *and* $e^{**} = e^s > 0$.

(ii) $\beta^{**} > 0$, $\rho^{**} \geq 0$, $0 \leq \kappa_\iota^{**} < \kappa_\varepsilon^{**}$, *and* $e^{**} > 0$; *if* $\rho^{**}, \kappa_\iota^{**} > 0$, *then* $0 < \kappa_\iota^{**} < \kappa_\varepsilon^{**} < l$.

In particular, if $\phi_\varepsilon' \geq 0$ and $\phi_b' \leq 0$, i.e., $(1-\theta)q \geq 1-p$ and $\frac{\phi_n'}{\phi_n} \leq \frac{\psi'}{1-\psi}$ for all $e \in [0,1]$, then $\kappa_\iota^{**} > 0$ and thus $\beta^{**} > 0$. Furthermore, for p close to 1, $\beta^{**} > l - \kappa_\varepsilon^{**}$.

Perhaps surprisingly, Proposition 3 states that the first-best relational contract may not employ the subjective signal y_t even if it is contractible. At first, this seems to contradict the informativeness principle (Holmstrom, 1979), which suggests that y_t should be employed no matter how noisy it is, because it contains information about e_t beyond x_t. However, such an insight relies on the solution being interior so that local improvement is feasible. If the optimal spot contract is a boundary solution, i.e., $k_\iota^* = 0$, then an extra informative signal may not be used. In this sense, Proposition 3 complements the informativeness principle.

Intuitively, if the nonnegativity constraint of k_ι is binding in the optimal spot contract, a negative k_ι would have been imposed to enhance incentives without such a constraint. To mitigate the distortion caused by the constraint, the principal raises the premium such that the agent has higher marginal utility and is thus more sensitive to incentives. But since the agent pays premium in any case, a higher premium will harm risk sharing, and a positive penalty will aggravate this issue. If y_t is relatively noisy, the negative effect on risk sharing may outweigh the value of additional information and thus using y_t is suboptimal.

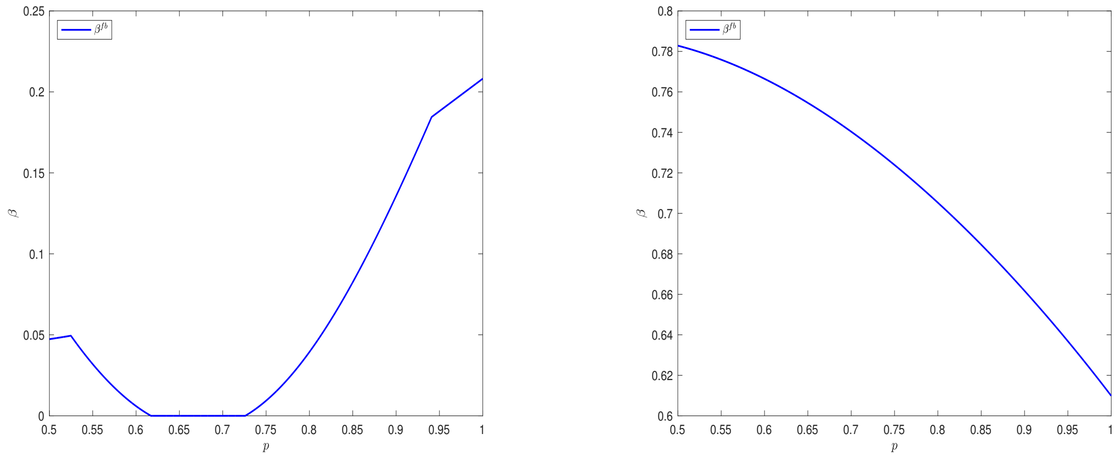

Figure 2: **The First-Best Relational Contract.** This figure compares the penalty level of the first-best relational contract of two examples for $p \in [0.5, 1]$. Both examples assume that $u(w) = \log(w)$, $h(e) = e^2/10$, $w = 5$, $l = 1$, $\theta = 0.75$, and $q = 0.6$. However, the left panel assumes that $\psi(e) = 0.15e$, whereas the right panel assumes that $\psi(e) = e$. Thus, the right panel has a higher value of $\left|\frac{\phi_g'}{\phi_g} - \frac{\phi_b'}{\phi_b}\right|$ equal to $\frac{1}{e(1-e)}$, whereas the value of the left panel is equal to $\frac{1}{e(1-0.15e)}$.

Figure 2 illustrates such an observation by comparing two different numerical examples, which only differ in the functional form of ψ. It is easy to show that for any e, the left panel of Figure 2 has a lower value of $\left|\frac{\phi_g'}{\phi_g} - \frac{\phi_b'}{\phi_b}\right|$ than the right panel; that is, y_t is less informative in the left panel. Consequently, signal y_t is not used by the first-best relational contract in some region of p in the left panel, whereas it is used over the domain of p in the right panel. Furthermore, in the right panel, the penalty level β is decreasing in the informativeness of the objective signal, which is measured by p. That is, the objective and subjective signals, more precisely, the explicit incentive (κ) and the implicit incentive (β) can be substitutes. On the other hand, they can instead be complements, as depicted in the left panel. To see the intuition, recall that when both signals are relatively noisy, a higher premium is desired to provide incentives by leveraging the wealth effect, thereby harming risk sharing; in turn, the implicit incentive will be dampened or even prohibited. As the objective signal becomes more informative, incentive provision is more efficient, and thus, a lower premium is charged, allowing for a higher penalty level to strengthen incentives. In contrast, when the subjective signal is relatively informative, the penalty itself can provide sufficient incentives, and will be substituted by the explicit incentive as the objective signal becomes more informative. As we shall discuss later, this insight turns out to be the driven force of our main results. A similar insight has been highlighted by Baker et al. (1994), where the agents are risk neutral and thus there is no wealth effect.

Analogous to the spot contract, when $\phi_\varepsilon' \geq 0$ and $\phi_b' \leq 0$, $\kappa_\iota^{**} > 0$. Intuitively, $\phi_\varepsilon' \geq 0$ and $\phi_b' \leq 0$ means that p, q and ψ' are relatively large. In other words, both the objective and subjective signals are relatively informative, and thus, the principal can better monitor the agent. This in turn leads to more insurance, i.e., $\kappa_\iota^{**} > 0$. In addition, as $p \to 1$, a low effort will be easily detected when there is a loss, and thus, a good objective signal ($x_t = \varepsilon$) is more indicative of a high effort than a bad subjective signal ($y_t = b$), leading to a higher payoff under $x_t = \varepsilon$, that is, $\beta^{**} > l - \kappa_\varepsilon^{**}$.

Apparently, if the first-best relational contract coincides with the optimal spot contract, then it is always self-enforcing since $\beta = 0$. In contrast, if the two contracts are distinct, the former will yield higher surplus than the latter due to the value of additional information. Since the agent's reneging temptation is bounded, if he is sufficiently patient, the first-best relational contract will be self-enforcing. By contrast, if the (DE-A) constraint is binding, then the resultant relational contract is the second-best and is solved similarly with (DE-A) binding such that $\gamma > 0$. In summary, we have the following proposition.

Proposition 4. *There exists some $\delta^* < 1$ such that for any $\delta \in (\delta^*, 1)$, the (DE-A) constraint is slack; thus, the optimal relational contract coincides with the first-best relational contract. When the (DE-A) constraint is binding, we have $\beta^* > 0$, $\rho^* \geq 0$, $0 \leq \kappa_\iota^* < \kappa_\varepsilon^*$, and $e^r > 0$; if further $\rho^*, \kappa_\iota^* > 0$, then we have $0 < \kappa_\iota^* < \kappa_\varepsilon^* < l$.*

If, in particular, δ is too low, such that no self-enforcing relational contract with a positive penalty level exists, then the principal offers the optimal spot contract to the agent.

4.1 The Impacts of Signal Informativeness

Our primary goal is to study how the informativeness of the subjective and objective signals affects the structure of optimal relational contract and social welfare. First, we examine the impact of the informativeness of the subjective signal. For ease of exposition, we parameterize $\psi(e)$ by introducing an implicit variable σ with a domain Σ, such that ψ can be rewritten as $\psi(e, \sigma)$. We assume that $\psi(e, \sigma)$ is smooth with $\psi_e, \psi_\sigma > 0$ on $[0, 1] \times \Sigma$.

It is instructive to relate the parameter σ to the informativeness of the subjective signal. The lemma below offers a sufficient condition for the informativeness of y_t to increase in σ.

Lemma 5. *Suppose $\psi_{e\sigma}\psi \geq \psi_e\psi_\sigma$ on $[0, 1] \times \Sigma$, then for any $\sigma_1 > \sigma_2$ and $e > 0$,*

$$\left| \frac{\phi_g'(e, \sigma_1)}{\phi_g(e, \sigma_1)} - \frac{\phi_b'(e, \sigma_1)}{\phi_b(e, \sigma_1)} \right| \geq \left| \frac{\phi_g'(e, \sigma_2)}{\phi_g(e, \sigma_2)} - \frac{\phi_b'(e, \sigma_2)}{\phi_b(e, \sigma_2)} \right|.$$

Thus, y_t is more informative with the monitoring technology σ_1 than with σ_2.

To interpret the sufficient condition in Lemma 5, note that it is equivalent to

$$\frac{\psi_{e\sigma}}{\psi_e} / \frac{\psi_\sigma}{\psi} \geq 1 \text{ on } [0,1] \times \Sigma. \tag{9}$$

Note that on the left hand side of the inequality, the numerator is the percentage change in the marginal effect of effort (in terms of generating a good signal) caused by the change in monitoring technology, and the denominator is the percentage change in the probability of a good signal, which could be viewed as the percentage change in monitoring technology. In this sense, the above ratio can be regarded as *the monitoring technology elasticity of the marginal effect of effort*. As such, Lemma 5 indicates that if the marginal effect of effort is elastic with respect to monitoring technology ranked by σ, then the subjective signal y_t is more informative with a monitoring technology associated with a higher σ.

Now, we can be more precise about the impacts of the informativeness of the subjective signal. The next proposition shows that if the marginal effect of effort is elastic, then welfare is higher under a better monitoring technology (with a higher σ). This implies that welfare is higher under a more informative subjective signal.

Proposition 5. *Suppose (9) holds, then the agent's payoff (social welfare) from the relational contract, V^r, is increasing in σ. That is, if the marginal effect of effort is elastic with respect to monitoring technology, then V^r is higher when the subjective signal y_t is more informative.*

Intuitively, the more informative the subjective signal, the better the implicit incentive and thus the more additional surplus a relational contract can generate than a spot contract, thereby reducing the agent's reneging temptation. Thus, irrespective of whether the (DE-A) constraint is binding, an optimal relational contract is more socially beneficial under a more informative subjective signal.

In contrast, the welfare implication of the objective signal can be non-monotonic. On the one hand, a more informative objective signal can make a relational contract more efficient. On the other hand, it can also increase the return of the fallback, that is, the optimal spot contract, thereby tightening the (DE-A) constraint. To be precise, by the envelope theorem, we have $dV^r/dp = \partial \mathcal{L}/\partial p$, which is equal to

$$\begin{aligned}&\left(1+\frac{\gamma\delta}{1-\delta}\right)\left(\frac{\partial \phi_\iota}{\partial p}u_\iota + \frac{\partial \phi_\varepsilon}{\partial p}u_\varepsilon\right) - \lambda\left(\frac{\partial \phi_\iota}{\partial p}\kappa_\iota^* + \frac{\partial \phi_\varepsilon}{\partial p}\kappa_\varepsilon^*\right) + \mu\left(\frac{\partial \phi'_\iota}{\partial p}u_\iota + \frac{\partial \phi'_\varepsilon}{\partial p}u_\varepsilon\right) - \frac{\gamma\delta}{1-\delta}\frac{dV^s}{dp} \\ &= (1-e^r)(\kappa_\varepsilon^* - \kappa_\iota^*)\left[\lambda - \frac{u_\varepsilon - u_\iota}{\kappa_\varepsilon^* - \kappa_\iota^*}\left(1 - \frac{\mu}{1-e^r}\right)\right] \underbrace{-\frac{\gamma\delta}{1-\delta}\left[\frac{dV^s}{dp} + (1-e^r)(u_\varepsilon - u_\iota)\right]}_{\text{The impact on the (DE-A) constraint}}. \end{aligned} \tag{10}$$

The terms above the bracket is the marginal effect of p on the (DE-A) constraint. It follows from Propositions 2 and 4 that these terms are non-positive, being strictly negative if $\gamma > 0$. If (DE-A) is slack, i.e., $\gamma = 0$, then we can prove similar to Proposition 2 that $dV^r/dp > 0$. That is, the first-best relational contract will yield higher social welfare when the objective signal becomes more precise. By contrast, if (DE-A) is binding, then the total effect of p is ambiguous. In particular, if (DE-A) binds tightly, then the negative effect of p may outweigh the positive effect. Consequently, a more precise objective signal may lead to a less efficient second-best relational contract, or even to non-existence of self-enforcing relational contract. Similarly, we have $dV^r/dq = \partial\mathcal{L}/\partial q$, which is equal to

$$e^r(1-\theta)(\kappa_\varepsilon^* - \kappa_\iota^*)\left[\frac{u_\varepsilon - u_\iota}{\kappa_\varepsilon^* - \kappa_\iota^*}\left(1+\frac{\mu}{e^r}\right) - \lambda\right] - \frac{\gamma\delta}{1-\delta}\left[\frac{dV^s}{dq} - e^r(1-\theta)(u_\varepsilon - u_\iota)\right]. \tag{11}$$

Again, if (DE-A) is slack, then it can be shown that the first-best relational contract will be more efficient as q increases; otherwise, the sign of (11) is also uncertain.

Since V^r increases with the objective signal precision whenever (DE-A) is slack, we now discuss how parameters δ, p, and q affect the slackness of (DE-A). Proposition 4 guarantees the slackness of (DE-A) for sufficiently large δ. In terms of p and q, there are two cases. On the one hand, if the optimal spot contract is relatively inefficient, for example when both p and q are close to 0.5, then (DE-A) is slack if δ is not low. On the other hand, conceivably, when the objective signal is already relatively precise, the optimal spot contract cannot be much improved by only a more precise objective signal; instead, a relational contract could yield sufficiently more surplus than the optimal spot contract, so that (DE-A) will be slack. In summary, for relatively big δ, the first-best relational contract is self-enforcing when the objective signal is either sufficiently noisy or sufficiently precise. Therefore, the agent's payoff from the relational contract, V^r, will be increasing in such region of p and q. Formally,

Proposition 6. *The agent's payoff V^r is increasing in both p and q for any $\delta \in (\delta^*, 1)$. Moreover, there exists some $\delta^{**} \leq \delta^*$, such that for any $\delta \in (\delta^{**}, 1)$, (DE-A) is slack and thus V^r is increasing in both p and q when (p, q) is in the right neighborhood of $(0.5, 0.5)$, or when p is in the left neighborhood of 1.*

The second part of Proposition 6 results from the fact that when either (p, q) is close to $(0.5, 0.5)$, or p is close to 1, $k_\iota^* > 0$. It follows from Proposition 3 that in the first-best relational contract $\beta^{**} > 0$. This implies that the first-best relational contract can generate sufficiently more surplus than the optimal spot contract, thereby being self-enforcing, if δ is relatively high. Moreover, because when $\delta > \delta^*$, the first-best relational contract is always self-enforcing, it is readily confirmed that $\delta^{**} \leq \delta^*$.

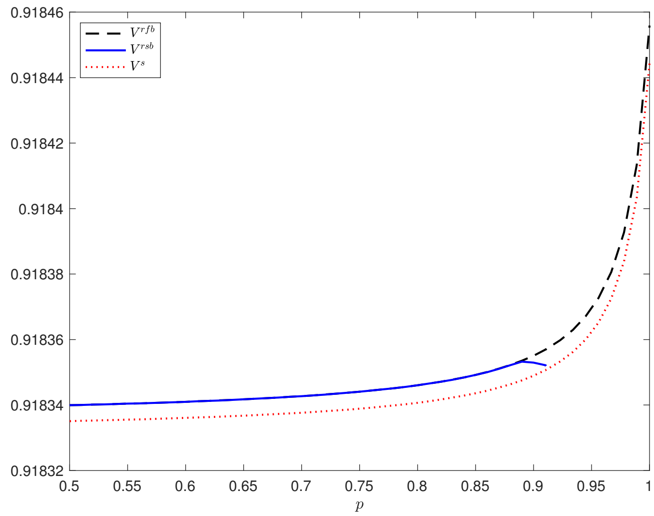

Figure 3: **The Welfare Implication of Objective Signal precision.** This figure illustrates the agent's payoff from, respectively, the first-best relational contract, the optimal relational contract, and the optimal spot contract, for $p \in [0.5, 1]$. The figure assumes that $u(w) = \log(w)$, $h(e) = 3e^2/2$, $\psi(e) = \max\{2e - 0.06, 0\}$, $w = 3$, $l = 0.5$, $\theta = 0.6$, $q = 0.9$, and $\delta = 0.9998$.

Proposition 6 and the above argument suggest that as the objective signal becomes more precise, there might successively exist four regions, such that the optimal relational contract first becomes more efficient since it is the first-best, then it becomes less efficient as it turns into the second-best with (DE-A) increasingly binding, and then no self-enforcing relational contract exists, and lastly self-enforcing restores and the optimal relational contract becomes increasingly efficient. It is likely that only a subset of these regions can co-exist given δ.

Figure 3 illustrates this observation by comparing the agent's payoff from, respectively, the first-best relational contract, the optimal relational contract, as well as the optimal spot contract, for some intermediate level of δ and $p \in [0.5, 1]$. As depicted, when p is sufficiently low, the first-best relational contract is self-enforcing, thus V^r is increasing in p. When p is relatively high, say around 0.85, the first-best relational contract is no longer self-enforcing; instead, the optimal relational contract is the second-best, while V^r is still increasing in p. It is worth noting, however, that when p is around 0.9, whereas the optimal relational contract is still self-enforcing, it yields lower social welfare as p increases in the region. This is because the fallback is relatively attractive and thus the agent faces an intense reneging temptation. In turn, a significant downward distortion has to be imposed on penalty level to maintain self-enforcing, leading to intensive-margin inefficiency. Finally, when p is sufficiently high, no self-enforcing relational contract exists since the fallback is too attractive, so the market returns to the less efficient optimal spot contract, leading to extensive-margin inefficiency. In summary, a more precise objective signal can reduce the insurance market efficiency.

5 Policy Implications

In the previous section, we characterized the optimal relational contract, and examined the impacts of the informativeness of the subjective and objective signals on the structure of the optimal relational contract and the insurance market efficiency. These results may have meaningful implications for the insurance market in the digital era.

First, our paper reveals when novel usage-based insurance contracts such as pay-how-you-drive plans are likely to emerge. Proposition 5 indicates that the more informative the subjective signal, the more efficient the relational contract. In contrast, a more informative objective signal may reduce the efficiency of such contract, particularly when the subjective signal is relatively noisy and the self-enforcing constraint is tightly biding. This implies that such novel relational insurance contracts are less likely to emerge in the insurance markets where telematics and data analytical technologies are underdeveloped (resulting in relatively noisy subjective signals), while the transportation infrastructure is well-established, such as widespread use of traffic cameras (resulting in relatively precise objective signals).

Second, our paper sheds light on the potential benefits from regulating these insurance policies. Such regulation could involve requiring insurance companies to be more transparent in their evaluation algorithms and pricing terms, thereby reducing disputes between insurers and policyholders. Additionally, it could also entail strengthening the enforceability of these informal contracts, thus deterring ex-post opportunistic behaviors. These measures can be socially beneficial in the following aspects.

From a short-run perspective, such regulation can mitigate the distortion in the design of the optimal relational contract due to the weak enforceability of the subjective signal. In particular, it can restore the optimal interplay between the objective and subjective signals, thereby improving the efficiency of the relational contract, or rendering a relatively efficient but previously not self-enforcing relational contract feasible.

From a long-run perspective, such regulation may also mitigate the distortion in the ex-ante investment in monitoring technology in insurance market. To see this, suppose before the insurer designs the contract, a social planner can choose p and σ through investment to maximize social welfare. The optimal choice depends on the marginal benefit of investment in each variable, which is derived in previous sections. As a reference point, we derive the

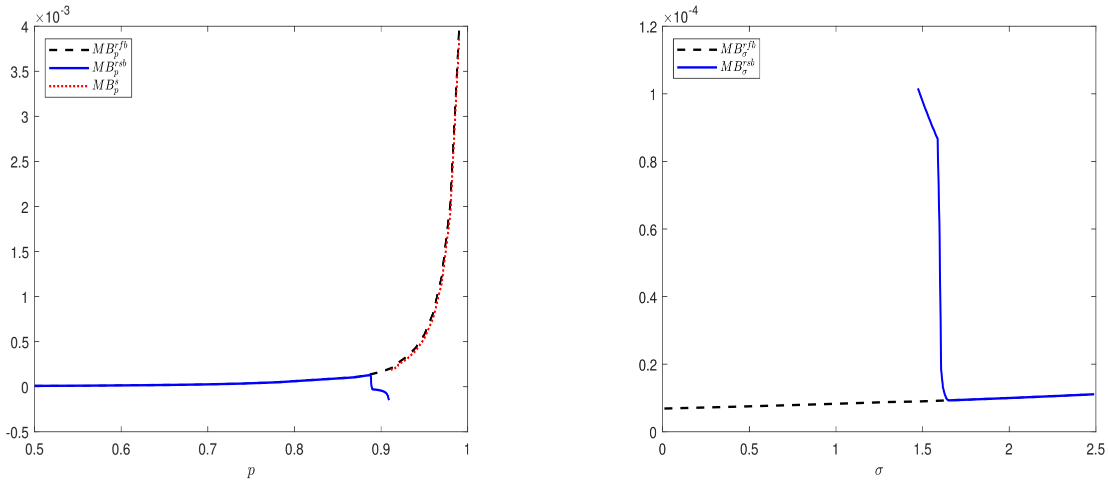

Figure 4: **The Marginal Benefits of Investment in the Objective and Subjective Signals.** This figure compares the marginal benefit of investment in, respectively, p and σ, between the optimal relational contract and the first-best relational contract. Both panels assume that $u(w) = \log(w)$, $h(e) = 3e^2/2$, $w = 3$ and $l = 0.5$, whereas the left penal assumes that $\psi(e) = \max\{2e - 0.06, 0\}$, $\theta = 0.6$, $q = 0.9$ and $\delta = 0.9998$, and the right panel assumes that $\psi(e) = \sigma e$, $\theta = 0.9$, $p = 0.8$, $q = 0.8$ and $\delta = 0.9995$.

marginal benefit of the first-best relational contract by simply substituting $\gamma = 0$ into the associated marginal benefit of the optimal relational contract. To illustrate, we consider for each variable a numerical example, and compare the marginal benefit between the optimal relational contract and the first-best relational contract in Figure 4. Obviously, the marginal benefit of the two cases coincide whenever the first-best relational contract is self-enforcing. In this case, the optimal relational contract leads to the efficient investment in both signals.

However, as shown in Figure 4, inefficient investment may happen for both the objective and subjective signals. The left panel depicts the marginal benefit of investment in p. Note that the marginal benefit of the first-best relational contract MB_p^{rfb} is increasing and positive in the domain of p, whereas that of the optimal relational contract MB_p^{rsb} is positive only for p lower than around 0.85. For higher p, either MB_p^{rsb} is negative, or no relational contract is self-enforcing such that the equilibrium contract is the optimal spot contract which yields a lower marginal benefit than MB_p^{rfb}. Suppose the marginal cost curve is upward sloping and single-crossing MB_p^{rfb} from below. Hence, unless the marginal cost curve intersects MB_p^{rfb} at some point where $MB_p^{rfb} = MB_p^{rsb}$, there will be underinvestment in the objective signal.

More interestingly, the right panel of Figure 4 suggests that both underinvestment and overinvestment in the subjective signal are possible. Note that the marginal benefit of the first-best relational contract MB_σ^{rfb} is increasing and positive in the domain of σ. In contrast, when σ is relatively low, that is, when the subjective signal is relatively noisy, the marginal benefit of the optimal relational contract MB_σ^{rsb} is zero, since no relational contract is self-enforcing. By contrast, for σ around 1.5, notably, $MB_\sigma^{rsb} > MB_\sigma^{rfb}$, because in this region, the self-enforcing constraint is tightly binding, and thus, the marginal improvement of the subjective signal can remarkably relax this constraint, leading to higher social welfare. Thus, if the marginal cost curve is relatively steep so that it intersects MB_σ^{rfb} at some low σ, then the optimal relational contract may feature underinvestment—no investment precisely—in the subjective signal. By contrast, if the intersection is at some point around 1.5, then the optimal relational contract may feature overinvestment in the subjective signal.

In practice, however, it is common for the government to invest in the objective signal, while the firms invest in the subjective signal. This division of public and private investment may arguably result in greater inefficiencies in both types of signals than in the above case. For example, a relatively competent government might overinvest in objective signals thereby crowding out private investment in subjective signals, whereas a less competent one might stimulate private investment excessively. Thus, the aforementioned regulatory policies can enhance social welfare by reducing distortions in investment in monitoring technology.

6 Conclusion and Discussion

In this paper, we explore the combined use of objective and subjective performance measures in insurance contracts under moral hazard. In each period, a competitive insurer observes an objective and a subjective signal about the insured's behavior and use them as, respectively, the explicit and implicit incentive components of the contract. We show that the implicit incentive component may not be used even if it is enforceable when the subjective signal is relatively noisy. Moreover, the objective and subjective signals can be both substitutes and complements. As the signals become more precise, they may impose qualitatively different impacts on the insurance contract and welfare: Whereas a more precise subjective signal can always improve the insurance market efficiency, the welfare implication of the objective signal precision can be non-monotonic. In particular, if a more precise objective signal leads to a sufficiently attractive fallback of the insurance contract, it may reduce the efficiency of the contract, or even make it infeasible to employ the subjective signal. Hence, our results have meaningful implications for the burgeoning usage-based insurance (UBI) market. They allow us to explain under what conditions these novel insurance policies would emerge. Moreover, they suggest that the regulation of UBI markets can potentially mitigate the distortions in the design of UBI contracts and the ex-ante investment in related monitoring technologies.

Our paper also sheds light on how digital technologies may change the landscape of the insurance market. With the increasing prevalence of UBI, the precision of UBI signal becomes paramount for insurance contracts, as it can significantly enhance contract efficiency. While traditional insurance companies have a number of comparative advantages such as consumer base and financial strength, they may lag behind in digital technology compared to smart car makers such as Tesla. Thus, we anticipate an influx of new market entrants, particularly smart car makers, into the insurance sector. This trend is likely to foster the emergence of joint ventures between insurance companies and car makers, as well as spur the growth of innovative insurance companies.

In many instances, disputes between insurers and policyholders emerge when the driver-assistance system autonomously executes driving decisions, such as harsh braking, yet the driver is still held fully responsible for the outcomes. From a broader perspective, our paper underscores the emerging challenges associated with the disconnection of act and liability in the digital era. That is, increasingly, decisions are made by artificial intelligence (AI), yet individuals bear the consequences. This disconnection is not limited to driving but extends to AI-assisted medical treatments and other critical scenarios where AI plays a pivotal role. Oftentimes, those accountable for certain actions cannot simply override AI-driven decisions. How can we resolve such conflicts technologically and institutionally? How can we better exploit digital technologies to improve the wellbeing of the whole society? We wish to see more extensive research on this important issue.

A Appendix

A Proofs for Section 3

Proof of Lemma 1

Proof. From basic calculation, we have

$$\frac{\phi_n'}{\phi_n} = \frac{1}{e} > 0,$$
$$\frac{\phi_\iota'}{\phi_\iota} = \frac{(1-\theta)(1-q)-p}{(1-e)p+e(1-\theta)(1-q)} < 0,$$
$$\frac{\phi_\varepsilon'}{\phi_\varepsilon} = \frac{(1-\theta)q-(1-p)}{e(1-\theta)q+(1-e)(1-p)}.$$

The second inequality follows from $(1-\theta)(1-q) < 0.5 < p$. To show that MLRP holds, we consider two cases. First, when $(1-\theta)q-(1-p) \geq 0$, we have $\frac{\phi_\iota'}{\phi_\iota} < 0 \leq \frac{\phi_\varepsilon'}{\phi_\varepsilon}$. Note that

$$\frac{\phi_\varepsilon}{\phi_\varepsilon'} - \frac{\phi_n}{\phi_n'} = \frac{1-p}{(1-\theta)q-(1-p)} > 0,$$

and thus, $\frac{\phi_\varepsilon'}{\phi_\varepsilon} < \frac{\phi_n'}{\phi_n}$. Second, when $(1-\theta)q-(1-p) \leq 0$, we have $\frac{\phi_\varepsilon'}{\phi_\varepsilon} \leq 0 < \frac{\phi_n'}{\phi_n}$. Note that

$$\frac{\phi_\iota}{\phi_\iota'} - \frac{\phi_\varepsilon}{\phi_\varepsilon'} = \frac{p}{(1-\theta)(1-q)-p} - \frac{1-p}{(1-\theta)q-(1-p)} > 0,$$

and thus, $\frac{\phi_\iota'}{\phi_\iota} < \frac{\phi_\varepsilon'}{\phi_\varepsilon}$. In summary, $\frac{\phi_\iota'}{\phi_\iota} < \frac{\phi_\varepsilon'}{\phi_\varepsilon} < \frac{\phi_n'}{\phi_n}$, with $\frac{\phi_\iota'}{\phi_\iota} < 0 < \frac{\phi_n'}{\phi_n}$. It is also readily confirmed that $\frac{\phi_\varepsilon'}{\phi_\varepsilon} \to \frac{\phi_n'}{\phi_n}$ as $p \to 1$, and that $\frac{\phi_\iota'}{\phi_\iota} \to \frac{\phi_\varepsilon'}{\phi_\varepsilon}$ as $p, q \to 0.5$. Thus, the lemma is proven. □

Proof of Lemma 2

Proof. Note that the first order condition of e is given by

$$-\lambda(\phi_\iota' k_\iota + \phi_\varepsilon' k_\varepsilon) - \mu h''(e) = 0. \tag{A.1}$$

If $\lambda = 0$, then $\mu = 0$ since $h'' > 0$. This contradicts (3) and (4) since $u' > 0$; thus, $\lambda > 0$. It follows that (IR-P) is binding; thus, $r^* > 0$, i.e., $\nu_r = 0$, and we rule out that $k_\iota^* = k_\varepsilon^* = 0$.

Next, we prove that $\mu > 0$. First, we show that $\mu \neq 0$. Suppose not, then we consider three cases. If $k_\iota^*, k_\varepsilon^* > 0$, i.e., $\nu_\iota = \nu_\varepsilon = 0$, then (2) to (4) imply that $k_\iota = k_\varepsilon = l$. But (A.1) then becomes $-\lambda(\phi_\iota' + \phi_\varepsilon')l = \lambda\phi_n' l > 0$, a contradiction. If $k_\iota^* > 0$ and $k_\varepsilon^* = 0$, i.e., $\nu_\iota = 0$ and $\nu_\varepsilon > 0$, then $u_\iota' < u_\varepsilon'$ since $u'' < 0$. But (3) and (4) imply that $u_\iota' > u_\varepsilon'$, a contradiction. The case that $k_\iota^* = 0$ and $k_\varepsilon^* > 0$ leads to an analogous contradiction. Therefore, $\mu \neq 0$.

Now suppose $\mu < 0$, then we consider three cases. If $k_\iota^*, k_\varepsilon^* > 0$, i.e., $\nu_\iota = \nu_\varepsilon = 0$, then by (2) to (4) and Lemma 1, we have $k_\iota > k_\varepsilon > l$ and $u_\iota > u_\varepsilon > u_n$. When $(1-\theta)q-(1-p) < 0$, i.e., $\phi_\varepsilon' < 0$, we can construct a utility level $\hat{u}$ such that $u_\iota > u_\varepsilon > \hat{u} > u_n$. Then, we have

$$\phi_\iota' u_\iota + \phi_\varepsilon' u_\varepsilon + \phi_n' u_n - h'(e^s) < \phi_\iota' \hat{u} + \phi_\varepsilon' \hat{u} + \phi_n' \hat{u} - h'(e^s) = -h'(e^s) < 0.$$

It follows from (IC-A) that $e^s = 0$. But then the optimal contract is clearly full-insurance, i.e., $u_\iota = u_\varepsilon = u_n$, a contradiction. When $(1-\theta)q-(1-p) \geq 0$, i.e., $\phi_\varepsilon' \geq 0$, we can construct a $\hat{u}$ such that $u_\iota > \hat{u} > u_\varepsilon > u_n$, thereby arriving at an analogous contradiction. If $k_\varepsilon^* = 0$, i.e., $\nu_\varepsilon > 0$, then $u_n' < u_\varepsilon'$ due to that $u'' < 0$. But since $\mu < 0$, by Lemma 1, (2) and (4) imply that $u_n' > u_\varepsilon'$, a contradiction. Analogously, we can rule out that $k_\iota^* = 0$. Therefore, we must have $\mu > 0$.

Finally, if $k_\varepsilon^* = 0$, i.e., $\nu_\varepsilon > 0$, then $k_\iota^* > 0$; thus, by $u'' < 0$, we have $u_\iota' < u_\varepsilon'$. Since $\mu > 0$, (3) and (4) imply that $u_\iota' > u_\varepsilon'$, a contradiction. Thus, $\nu_\varepsilon = 0$, and the lemma is proven. □

Proof of Proposition 1

Proof. By Lemma 2, we have $k_\varepsilon^* > 0$. If $k_\iota^* = 0$, then $k_\iota^* < k_\varepsilon^*$. If $k_\iota^* > 0$, i.e., $\nu_\iota = 0$, then by Lemma 1 and (2) to (4), $0 < k_\iota^* < k_\varepsilon^* < l$. Suppose $\phi_\varepsilon' \geq 0$, then we must have $k_\iota^* > 0$; otherwise, (A.1) implies that $-\lambda\phi_\varepsilon' k_\varepsilon^* - \mu h'' = 0$, a contradiction. Then, we prove that $k_\varepsilon^* < l$ if $k_\iota^* = 0$. Suppose $k_\varepsilon^* \geq l$, then we have $u_\varepsilon' \leq u_n' < u_\iota'$. Then by (2) and (4), we have

$$\nu_\iota \geq \lambda\mu\phi_n \left(\frac{\phi_n'}{\phi_n} - \frac{\phi_\varepsilon'}{\phi_\varepsilon}\right) / \left(1 + \mu\frac{\phi_\varepsilon'}{\phi_\varepsilon}\right) = \mu u_\varepsilon' \phi_n \left(\frac{\phi_n'}{\phi_n} - \frac{\phi_\varepsilon'}{\phi_\varepsilon}\right).$$

Meanwhile, by (3) and (4), we have

$$\nu_\iota < \lambda\mu\phi_\iota \left(\frac{\phi_\varepsilon'}{\phi_\varepsilon} - \frac{\phi_\iota'}{\phi_\iota}\right) / \left(1 + \mu\frac{\phi_\varepsilon'}{\phi_\varepsilon}\right) = \mu u_\varepsilon' \phi_\iota \left(\frac{\phi_\varepsilon'}{\phi_\varepsilon} - \frac{\phi_\iota'}{\phi_\iota}\right).$$

Combining the above inequalities and noting that $\mu u_\varepsilon' > 0$, we have

$$\phi_n \left(\frac{\phi_n'}{\phi_n} - \frac{\phi_\varepsilon'}{\phi_\varepsilon}\right) < \phi_\iota \left(\frac{\phi_\varepsilon'}{\phi_\varepsilon} - \frac{\phi_\iota'}{\phi_\iota}\right).$$

Rearranging and noting that $\phi_n' + \phi_\iota' = -\phi_\varepsilon'$ and $\phi_n + \phi_\iota = 1 - \phi_\varepsilon$, we have $\phi_\varepsilon' > 0$. But now we have $k_\iota^* > 0$ by the above argument, a contradiction. Thus, we always have $k_\iota^* < k_\varepsilon^* < l$. Note that the first order condition of the agent's effort can be rewritten as

$$\phi_n'(u_n - u_\varepsilon) - \phi_\iota'(u_\varepsilon - u_\iota) - h'(e^s) = 0.$$

It follows that $e^s > 0$. Finally, by Lemma 1, if $p, q \to 0.5$, then $\frac{\phi_\iota'}{\phi_\iota} \to \frac{\phi_\varepsilon'}{\phi_\varepsilon}$. Thus, by (2) to (4) and (A.1), we have $0 < k_\iota^* = k_\varepsilon^* < l$. Therefore, the proposition is proven. □

Proof of Corollary 1

Proof. Statements (i) to (iii) follows from Proposition 1. To prove statement (iv), note that

$$\lim_{\theta\to 1}\frac{\phi_\iota'}{\phi_\iota}=\lim_{\theta\to 1}\frac{\phi_\varepsilon'}{\phi_\varepsilon}=-\frac{1}{1-e^s}<0<\lim_{\theta\to 1}\frac{\phi_n'}{\phi_n}.$$

It follows from (2) to (4) and Proposition 1 that $0<\lim_{\theta\to 1}k_\iota^*=\lim_{\theta\to 1}k_\varepsilon^*<l$. This implies that for θ close to 1, $\nu_\iota=0$. Applying the implicit function theorem to (2) to (4), we have that λ and μ are continuously differentiable in θ. Note too that

$$\frac{\partial}{\partial\theta}\frac{\phi_\iota'}{\phi_\iota}=-\frac{p(1-q)}{\phi_\iota^2}\quad\text{and}\quad\frac{\partial}{\partial\theta}\frac{\phi_\varepsilon'}{\phi_\varepsilon}=-\frac{(1-p)q}{\phi_\varepsilon^2}.$$

It follows that for θ close to 1, $\frac{\partial}{\partial\theta}\frac{\phi_\varepsilon'}{\phi_\varepsilon}<\frac{\partial}{\partial\theta}\frac{\phi_\iota'}{\phi_\iota}<0$. Then by (3) and (4), for θ close to 1,

$$\begin{aligned}\frac{du_\iota'}{d\theta}&=\frac{\frac{d\lambda}{d\theta}(1+\mu\frac{\phi_\iota'}{\phi_\iota})-\lambda(\frac{d\mu}{d\theta}\frac{\phi_\iota'}{\phi_\iota}+\mu\frac{\partial}{\partial\theta}\frac{\phi_\iota'}{\phi_\iota})}{(1+\mu\frac{\phi_\iota'}{\phi_\iota})^2},\\ \frac{du_\varepsilon'}{d\theta}&=\frac{\frac{d\lambda}{d\theta}(1+\mu\frac{\phi_\varepsilon'}{\phi_\varepsilon})-\lambda(\frac{d\mu}{d\theta}\frac{\phi_\varepsilon'}{\phi_\varepsilon}+\mu\frac{\partial}{\partial\theta}\frac{\phi_\varepsilon'}{\phi_\varepsilon})}{(1+\mu\frac{\phi_\varepsilon'}{\phi_\varepsilon})^2}.\end{aligned}$$

It follows from continuity and above that for θ close to 1,

$$\frac{d(u_\iota'-u_\varepsilon')}{d\theta}\approx-\frac{\lambda\mu(\frac{\partial}{\partial\theta}\frac{\phi_\iota'}{\phi_\iota}-\frac{\partial}{\partial\theta}\frac{\phi_\varepsilon'}{\phi_\varepsilon})}{(1-\frac{\mu}{1-e^s})^2}<0.$$

This implies that within some left neighborhood of 1, $k_\varepsilon^*-k_\iota^*$ decreases and converges to 0 as $\theta\to 1$. Thus, the corollary is proven. □

Proof of Proposition 2

Proof. We first prove that $\frac{dV^s}{dp}>0$ and $\frac{dV^s}{dq}>0$. Note that

$$\begin{aligned}\frac{dV^s}{dp}=\frac{\partial\mathcal{L}}{\partial p}&=\frac{\partial\phi_\iota}{\partial p}u_\iota+\frac{\partial\phi_\varepsilon}{\partial p}u_\varepsilon-\lambda\left(\frac{\partial\phi_\iota}{\partial p}k_\iota^*+\frac{\partial\phi_\varepsilon}{\partial p}k_\varepsilon^*\right)+\mu\left(\frac{\partial\phi_\iota'}{\partial p}u_\iota+\frac{\partial\phi_\varepsilon'}{\partial p}u_\varepsilon\right)\\ &=(1-e^s)u_\iota-(1-e^s)u_\varepsilon-\lambda(1-e^s)k_\iota^*+\lambda(1-e^s)k_\varepsilon^*-\mu u_\iota+\mu u_\varepsilon\\ &=(1-e^s)\left[\lambda(k_\varepsilon^*-k_\iota^*)-(u_\varepsilon-u_\iota)\left(1-\frac{\mu}{1-e^s}\right)\right]\\ &=(1-e^s)(k_\varepsilon^*-k_\iota^*)\left[\lambda-\frac{u_\varepsilon-u_\iota}{k_\varepsilon^*-k_\iota^*}\left(1-\frac{\mu}{1-e^s}\right)\right]\\ &=(1-e^s)(k_\varepsilon^*-k_\iota^*)\left[u_\iota'\left(1+\mu\frac{\phi_\iota'}{\phi_\iota}\right)+\frac{\nu_\iota}{\phi_\iota}-\frac{u_\varepsilon-u_\iota}{k_\varepsilon^*-k_\iota^*}\left(1-\frac{\mu}{1-e^s}\right)\right]>0.\end{aligned}$$

The last equality is due to (3). The inequality is due to that $u'_\iota > \frac{u_\varepsilon - u_\iota}{k^*_\varepsilon - k^*_\iota}$ because $u'' < 0$ and $k^*_\varepsilon > k^*_\iota$, and that $\frac{\phi'_\iota}{\phi_\iota} > -\frac{1}{1-e_t}$. Similarly, we have

$$\begin{aligned}\frac{dV^s}{dq} = \frac{\partial \mathcal{L}}{\partial q} &= \frac{\partial \phi_\iota}{\partial q} u_\iota + \frac{\partial \phi_\varepsilon}{\partial q} u_\varepsilon - \lambda \left(\frac{\partial \phi_\iota}{\partial q} k^*_\iota + \frac{\partial \phi_\varepsilon}{\partial q} k^*_\varepsilon \right) + \mu \left(\frac{\partial \phi'_\iota}{\partial q} u_\iota + \frac{\partial \phi'_\varepsilon}{\partial q} u_\varepsilon \right) \\ &= -e^s(1-\theta)u_\iota + e^s(1-\theta)u_\varepsilon + \lambda e^s(1-\theta)(k^*_\iota - k^*_\varepsilon) + \mu(1-\theta)(u_\varepsilon - u_\iota) \\ &= e^s(1-\theta) \left[\lambda(k^*_\iota - k^*_\varepsilon) - (u_\iota - u_\varepsilon) \left(1 + \frac{\mu}{e^s} \right) \right] \\ &= e^s(1-\theta)(k^*_\varepsilon - k^*_\iota) \left[\frac{u_\varepsilon - u_\iota}{k^*_\varepsilon - k^*_\iota} \left(1 + \frac{\mu}{e^s} \right) - \lambda \right] \\ &= e^s(1-\theta)(k^*_\varepsilon - k^*_\iota) \left[\frac{u_\varepsilon - u_\iota}{k^*_\varepsilon - k^*_\iota} \left(1 + \frac{\mu}{e^s} \right) - u'_\varepsilon \left(1 + \mu \frac{\phi'_\varepsilon}{\phi_\varepsilon} \right) \right] > 0.\end{aligned}$$

The inequality is because $u'_\varepsilon < \frac{u_\varepsilon - u_\iota}{k^*_\varepsilon - k^*_\iota}$ and $\frac{\phi'_\varepsilon}{\phi_\varepsilon} < \frac{\phi'_n}{\phi_n} = \frac{1}{e^s}$.

Second, we prove that $\frac{dV^s}{d\theta} > 0$. Note that $\frac{dV^s}{d\theta}$ is equal to

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \theta} &= \frac{\partial \phi_n}{\partial \theta} u_n + \frac{\partial \phi_\iota}{\partial \theta} u_\iota + \frac{\partial \phi_\varepsilon}{\partial \theta} u_\varepsilon - \lambda \left(\frac{\partial \phi_\iota}{\partial \theta} k^*_\iota + \frac{\partial \phi_\varepsilon}{\partial \theta} k^*_\varepsilon \right) + \mu \left(\frac{\partial \phi'_n}{\partial \theta} u_n + \frac{\partial \phi'_\iota}{\partial \theta} u_\iota + \frac{\partial \phi'_\varepsilon}{\partial \theta} u_\varepsilon \right) \\ &= (e^s + \mu)[u_n - (1-q)u_\iota - qu_\varepsilon] + \lambda e^s[(1-q)k^*_\iota + qk^*_\varepsilon] \\ &> (e^s + \mu)[u_n - (1-q)u_\iota - qu_\varepsilon] \\ &> (e^s + \mu)(u_n - u_\varepsilon) > 0.\end{aligned}$$

Thus, the proposition is proven. □

B Proofs for Section 4

Proof of Lemma 4

Proof. Note that the first order condition of e is given by

$$\lambda[\phi'_b \beta - \phi'_\iota \kappa_\iota - \phi'_\varepsilon \kappa_\varepsilon] + \mu[\phi''_g(u_g - u_b) - h''(e)] = 0. \tag{A.2}$$

If $\lambda = 0$, then $\mu = 0$ by the second order condition of the agent's problem. This contradicts (7) and (8) since $u' > 0$; thus, $\lambda > 0$, i.e., (IR-P) is binding, ruling out that $\kappa^*_\iota = \kappa^*_\varepsilon = 0$.

Next, we prove that $\mu > 0$. Suppose not, then $\mu \le 0$. It follows from Lemma 3 and (5) and (6) that $u'_g \ge u'_b$, meaning that $\beta \le 0$, a contradiction.

Finally, we prove that $\nu_\varepsilon = 0$. Suppose not, then $\nu_\varepsilon > 0$. Because κ^*_ι and κ^*_ε cannot both be zero, we have $\nu_\iota = 0$. It follows that $\kappa^*_\iota > \kappa^*_\varepsilon = 0$, meaning that $u'_\iota < u'_\varepsilon$. But since $\mu > 0$ and $\nu_\varepsilon > \nu_\iota = 0$, it follows from Lemma 1 and (7) and (8) that $u'_\iota > u'_\varepsilon$, a contradiction. Thus, the lemma is proven. □

Proof of Proposition 3

Proof. First, consider when $\beta^{**} = 0$. In this case, the fist-best relational contract coincides with the optimal spot contract. It follows from Proposition 1 that $\rho^{**} = r^* > 0$, $\kappa_\iota^{**} = k_\iota^* \geq 0$, $0 < \kappa_\varepsilon^{**} = k_\varepsilon^* < l$, and $e^{**} = e^s > 0$. Moreover, since $\beta^{**} = 0$, we have that at $\beta = 0$,

$$0 > \frac{\partial \mathcal{L}}{\partial \beta} = \lambda \phi_b - (\phi_b + \mu \phi_b') u_b' = \lambda \phi_b - (\phi_b + \mu \phi_b') u_n'.$$

The second equality is because $u_b' = u_n'$ when $\beta = 0$. Suppose $\kappa_\iota^{**} = k_\iota^* > 0$, i.e., $\nu_\iota = 0$, then by (2), we have $\lambda \phi_n - (\phi_n + \mu \phi_n') u_n' = 0$. It follows from Lemmas 2 and 3 that

$$0 > \phi_b \left[\lambda - \left(1 + \mu \frac{\phi_b'}{\phi_b} \right) u_n' \right] > \phi_b \left[\lambda - \left(1 + \mu \frac{\phi_n'}{\phi_n} \right) u_n' \right] = 0,$$

a contradiction. Thus, if $\beta^{**} = 0$, then $\kappa_\iota^{**} = k_\iota^* = 0$. In turn, if $\kappa_\iota^{**} > 0$, then $\beta^{**} > 0$.

Second, consider when $\beta^{**} > 0$. By Lemma 4 and (7) and (8), we have $0 \leq \kappa_\iota^{**} < \kappa_\varepsilon^{**}$. In particular, if $\rho^{**}, \kappa_\iota^{**} > 0$, i.e., $\nu_\rho = \nu_\iota = 0$, then (5) to (8) imply that $0 < \kappa_\iota^{**} < \kappa_\varepsilon^{**} < l$. It follows immediately that $e^{**} > 0$.

Now suppose $\phi_\varepsilon' \geq 0$ and $\phi_b' \leq 0$ for all $e \in [0, 1]$. Since $\kappa_\varepsilon^{**} > 0$, if $\kappa_\iota^{**} = 0$, it follows from (A.2) that $\frac{\partial \mathcal{L}}{\partial e} < 0$, meaning that $e^r = 0$, a contradiction. Finally, as $p \to 1$, $\frac{\phi_\varepsilon'}{\phi_\varepsilon} \to \frac{\phi_n'}{\phi_n} > \frac{\phi_b'}{\phi_b}$ due to Lemma 1. If in this case $\beta^{**} = 0$, then from the above we have $\kappa_\iota^{**} = k_\iota^* = 0$. But by Corollary 1, $\lim_{p \to 1} k_\iota^* > 0$, a contradiction. This implies that $\beta^{**} > 0$. It then follows from (5) and (8) that $\beta^{**} > l - \kappa_\varepsilon^{**}$. Thus, the proposition is proven. □

Proof of Proposition 4

Proof. We start with the first sentence of Proposition 4. If $\beta^{**} = 0$, then the statement holds automatically. If $\beta^{**} > 0$, then at β^{**}, we have

$$\frac{\partial \mathcal{L}}{\partial \beta} = \lambda \phi_b - (\phi_b + \mu \phi_b') u_b' = 0.$$

Fix e^{**} and λ and μ at the optimum. Since $u_b'' < 0$, $\frac{\partial \mathcal{L}}{\partial \beta} > 0$ for all $\beta \in [0, \beta^{**})$. This implies that the first-best relational contract provides the agent with a strictly higher payoff than the optimal spot contract. Since $u_g - u_b$ is bounded, as $\delta \to 1$, a relational contract will be strictly self-enforcing, i.e., the (DE-A) constraint is slack, if it takes the form of the first-best relational contract; clearly, it is optimal.

Then suppose the (DE-A) constraint is binding. In this case, $\beta^* > 0$ and $\rho^* \geq 0$. Using a similar argument to the proof of (ii) of Proposition 3, we have $0 \leq \kappa_\iota^* < \kappa_\varepsilon^*$, and thus, $e^r > 0$; if further $\rho^*, \kappa_\iota^* > 0$, then $0 < \kappa_\iota^* < \kappa_\varepsilon^* < l$. Thus, the proposition is proven. □

Proof of Lemma 5

Proof. By basic calculus, we have for any $e > 0$,

$$\frac{\partial}{\partial\sigma}\left|\frac{\phi_g'(e,\sigma)}{\phi_g(e,\sigma)} - \frac{\phi_b'(e,\sigma)}{\phi_b(e,\sigma)}\right| = \frac{\partial}{\partial\sigma}\frac{\psi_e(e,\sigma)}{\psi(e,\sigma)[1-\psi(e,\sigma)]}$$
$$= \frac{1}{\psi^2(1-\psi)}\left[\psi_{e\sigma}\psi - \psi_e\psi_\sigma + \frac{\psi}{1-\psi}\psi_e\psi_\sigma\right] > 0$$

if $\psi_{e\sigma}\psi \geq \psi_e\psi_\sigma$ on $[0,1]\times\Sigma$. Thus, the lemma is proven. □

Proof of Proposition 5

Proof. Note that $\frac{dV^r}{d\sigma}$ is equal to

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\sigma} &= \left(1+\frac{\gamma\delta}{1-\delta}\right)\left(\frac{\partial\phi_g}{\partial\sigma}u_g + \frac{\partial\phi_b}{\partial\sigma}u_b\right) + \lambda\frac{\partial\phi_b}{\partial\sigma}\beta + \mu\left(\frac{\partial\phi_g'}{\partial\sigma}u_g + \frac{\partial\phi_b'}{\partial\sigma}u_b\right)\\
&= \left[\left(1+\frac{\gamma\delta}{1-\delta}\right)\phi_n\psi_\sigma + \mu(\phi_n'\psi_\sigma + \phi_n\psi_{e\sigma})\right](u_g-u_b) - \lambda\phi_n\psi_\sigma\beta\\
&\geq \left[\left(1+\frac{\gamma\delta}{1-\delta}\right)\phi_n\psi_\sigma + \mu(\phi_n'\psi_\sigma + \phi_n\psi_{e\sigma})\right]u_g'\beta - \lambda\phi_n\psi_\sigma\beta\\
&\geq \left[\left(1+\frac{\gamma\delta}{1-\delta}\right)\phi_n\psi_\sigma + \mu(\phi_n'\psi_\sigma + \phi_n\psi_{e\sigma})\right]u_g'\beta - \left(1+\frac{\gamma\delta}{1-\delta}+\mu\frac{\phi_g'}{\phi_g}\right)u_g'\phi_n\psi_\sigma\beta\\
&= \frac{\mu\phi_n u_g'\beta}{\psi}\left[\psi_{e\sigma}\psi - \psi_e\psi_\sigma\right].
\end{aligned}$$

The first inequality is due to the concavity of $u(\cdot)$ and $u_g = u(w-\rho) \geq u(w-\rho-\beta) = u_b$. The second inequality follows from the substitution of (5) and noticing that $\frac{\gamma u_g'+\nu_\rho+\nu_\iota}{\phi_g} \geq 0$. The last equality uses the property that $\frac{\phi_n'}{\phi_n} - \frac{\phi_g'}{\phi_g} = -\frac{\psi_e}{\psi}$ in Lemma 3. By Lemma 4, $\mu > 0$. It follows that if (9) holds, then the last line is positive. That is, V^r is increasing in σ if $\beta > 0$. Thus, the proposition is proven. □

Proof of Proposition 6

Proof. We first prove that when (DE-A) is slack, V^r is increasing in both p and q. From the text, we have that when $\gamma = 0$,

$$\begin{aligned}
\frac{dV^r}{dp} &= (1-e^r)(\kappa_\varepsilon^* - \kappa_\iota^*)\left[\lambda - \frac{u_\varepsilon - u_\iota}{\kappa_\varepsilon^* - \kappa_\iota^*}\left(1-\frac{\mu}{1-e^r}\right)\right]\\
&= (1-e^r)(\kappa_\varepsilon^* - \kappa_\iota^*)\left[u_\iota'\left(1+\mu\frac{\phi_\iota'}{\phi_\iota}\right) + \frac{\nu_\iota}{\phi_\iota} - \frac{u_\varepsilon - u_\iota}{\kappa_\varepsilon^* - \kappa_\iota^*}\left(1-\frac{\mu}{1-e^r}\right)\right] > 0.
\end{aligned}$$

The inequality follows from the exactly same argument as in the proof of Proposition 2.

Similarly, we have that when $\gamma = 0$,

$$\frac{dV^r}{dq} = e^r(1-\theta)(\kappa_\varepsilon^* - \kappa_\iota^*)\left[\frac{u_\varepsilon - u_\iota}{\kappa_\varepsilon^* - \kappa_\iota^*}\left(1 + \frac{\mu}{e^r}\right) - \lambda\right]$$
$$= e^r(1-\theta)(\kappa_\varepsilon^* - \kappa_\iota^*)\left[\frac{u_\varepsilon - u_\iota}{\kappa_\varepsilon^* - \kappa_\iota^*}\left(1 + \frac{\mu}{e^r}\right) - u_\varepsilon'\left(1 + \mu\frac{\phi_\varepsilon'}{\phi_\varepsilon}\right)\right] > 0.$$

Suppose $p = q = 0.5$, then by Proposition 1, we have $0 < k_\iota^* = k_\varepsilon^* < 1$. It follows from Proposition 3 that in the first-best relational contract $\beta^{**} > 0$ if $p = q = 0.5$. Then by the proof of Proposition 4 and continuity, for sufficiently large $\delta < 1$, (DE-A) is slack if (p, q) is close to $(0.5, 0.5)$. Moreover, if p is close to 1, then by Corollary 1, $k_\iota^* > 0$. Again, we have $\beta^{**} > 0$ and thus (DE-A) is slack for large δ. Finally, by Proposition 4, (DE-A) is always slack when $\delta > \delta^*$, thus clearly we have $\delta^{**} \leq \delta^*$. Thus, the proposition is proven. □

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