Selling Signals

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Abstract

This paper studies a signaling model in which a strategic player can manipulate the cost of signaling. A seller chooses a price scheme for a good, and a buyer with a hidden type chooses how much to purchase as a signal to receivers. When receivers observe the price scheme, the seller charges monopoly prices, and the buyer purchases less than the first best. In contrast, when receivers do not observe the price scheme, the demand for signals is more elastic. In equilibrium, the seller charges lower prices, and the buyer purchases more than when receivers observe the price scheme; the highest types purchase more than the first best. The model suggests that price transparency benefits the seller but harms the buyer. The model can be applied to schools choosing tuition, retailers selling luxury goods and media companies selling advertisements.

Keywords: Signaling, Screening, Signal Jamming, Price Transparency

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1 Introduction

Signaling is prevalent in various markets. Whereas in classic signaling models, the sender's preference depends only on his intrinsic type, in many vertical markets in which signaling prevails, the signaling cost—thus the sender's preference—also depends on the choice made by an upstream strategic player. For example, when a student obtains education to signal his ability, the university sets the tuition; when a consumer purchases a luxury good to signal his wealth, the retailer chooses the price; when a firm incurs advertising expenses to signal its product's quality, the media company determines the costs of advertising messages.

A key observation is that since the signaling cost is endogenous, how receivers interpret and respond to the sender's signal depends on whether they observe the upstream player's choice. Consider a seller choosing the price of a good that can create signaling value for the customers, as in the above examples. How does receivers' information about the price affect the seller's pricing strategy? How does such information affect the degree of signaling and social welfare?

In this paper, we characterize the optimal price scheme for a seller facing a buyer who is endowed with a hidden type and chooses how much to purchase as a signal to receivers. The equilibrium depends critically on whether receivers observe the price scheme. When receivers observe the price scheme, the seller internalizes the buyer's signaling activity when screening the buyer, leading to a downward distortion in quantity. In contrast, when receivers do not observe the price scheme, the buyer is more sensitive to price changes, since receivers will attribute differences in choice only to buyer preference heterogeneity. This means that the demand for the good is more elastic, and thus, the seller lowers prices. In equilibrium, the buyer chooses a larger quantity and obtains higher utility, whereas the seller gains lower profits than when receivers observe the price scheme.

This paper has meaningful implications for the price transparency of signaling goods. In the case of job market signaling, our model suggests that education is more costly and students are worse off when employers observe the net prices for school than otherwise. This implies that policies that improve the transparency of the net prices at colleges and universities, e.g., U.S. Code § 1015a,[1] may *unintentionally* raise education expenses and harm students. This is because these policies allow schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers or financial aid.

[1]Since 2011, American colleges and universities have been required to provide reasonable estimates of the net prices, including tuition, miscellaneous fees and personal expenses, that students will pay for school. See "U.S. Code § 1015a - Transparency in college tuition for consumers" for details.

In addition, our model suggests that a signaling good yields higher profits if the price is more transparent. This is consistent with real-world business practices. For example, luxury brands, such as Louis Vuitton, Tiffany and Hermes, strive for a reputation of never or very rarely being on sale. These strategies help the sellers better commit to high prices, thereby maintaining the signaling values of luxury goods. In the advertising industry, the high costs of each year's Super Bowl commercials are widely reported, thereby enhancing the signaling value of these costly commercials; in China, the TV station CCTV broadcasts the auctions for its popular TV show commercials to accentuate their signaling values.

For expository purposes, we present our model à la Spence (1973) with a school selling productive education to a worker whose ability is privately known. As a reference point, we revisit Spence's game by assuming that competitive schools set tuition at marginal cost. In the least-cost separating equilibrium (i.e., the Riley outcome (Riley, 1979)), all worker types except the lowest choose more education than the first best, as they attempt to separate themselves from lower types. That is, signaling induces over-education.

Then, we consider the monopolistic school's pricing strategy and start with the case in which employers observe the tuition scheme. Following the screening literature, we focus on the seller-optimal equilibrium, in which all types except the highest choose less education than the first best. The downward distortion is due to screening. With lower marginal effort costs, a higher type has an incentive to mimic lower types. To incentivize truth-telling, the school leaves information rents to the worker, meaning that the marginal profit of education is less than the marginal social surplus. This induces the school to undersupply education.

While this mechanism is similar to screening models such as Mussa and Rosen (1978), our model also incorporates signaling, which can mitigate the downward distortion caused by screening. To illustrate, suppose that employers can observe the worker's ability, thereby eliminating signaling. When a higher type mimics a lower type, he not only incurs a lower cost than the latter but also obtains a higher wage due to higher ability. The second effect means that the worker can extract more information rents from the school. Therefore, the screening distortion is worse than when signaling is present.

Now, consider the case when employers do not observe the tuition scheme. We propose a new refinement, *quasi-divinity*, which selects the seller-optimal separating equilibrium (the associated Riley outcome), in which the school charges lower tuition and the worker chooses more education than when employers observe the tuition scheme. The difference is driven by a signal-jamming effect: the school jams the worker's signal when employers cannot observe the actual tuition scheme and thus infer the worker's ability based on a conjectured scheme.

To illustrate, suppose the school lowers tuition so that the worker chooses more education than in the initial state. When employers observe the tuition scheme, they cut wages, since any education level now corresponds to a lower-ability worker. This dampens the worker's demand for additional education. In contrast, when employers do not observe the tuition scheme, they do not adjust wages despite that tuition changes. Therefore, the demand for education is more elastic, making the price cut more profitable. In equilibrium, employers correctly anticipate the school's choice and offer lower wages since education is inflated. This reduces the worker's willingness to pay, and thus, the school gains lower profits.

Since the school is worse off when employers do not observe the tuition scheme, one may wonder why the school does not disclose tuition to employers. The reason is that the school cannot credibly announce the price absent intervention such as mandatory disclosure, since the school has an incentive to secretly cut prices. Such an observation may explain the fact that whereas the listed tuition at American colleges and universities is rising, these schools offer students various and inclusive forms of financial aid. The rationale is that employers cannot easily observe the details of such financial aid and thus do not know the actual cost of education. By raising the published tuition while simultaneously reducing the undisclosed net prices through stipends, schools persuade employers that their students are smarter than is actually the case, thereby allowing the schools to collect higher revenues from students.

Regarding welfare, we show that when signaling is sufficiently intense (e.g., when there is significant over-education in Spence's game), social welfare is higher when the tuition scheme is observed by employers than otherwise, since in the former case signaling mitigates the screening distortion to a large extent, whereas in the latter case, cheaper tuition induces many high types to overinvest in education. Moreover, when signaling is sufficiently intense, both cases yield higher social welfare than Spence's game in which schools are competitive. This implies that promoting competition in markets with signals is not necessarily socially beneficial, since it may lead to substantial wasteful signaling.

Finally, as extensions, we first consider the case in which the worker is productive even if he has no education (e.g., in Spence (1973), education is a pure signal). We show that our results are robust. Moreover, when employers observe the tuition scheme, the market may consist of a certification segment and an education segment: a lower interval of types pay a fixed fee for zero education, as if they were certified by the school as having the average productivity, whereas the higher types purchase education to signal their abilities. Next, we consider when the school maximizes a weighted average of its and the worker's payoffs. We show that our results remain unchanged, up to the relative Pareto weight of the two parties.

1.1 Related literature

This paper is most closely related to the literature on signaling. This paper contributes to signaling games by allowing a strategic player to affect the signaling cost. In classic signaling models (e.g., Spence 1973; Leland and Pyle 1977; Riley 1979; Milgrom and Roberts 1986; Bagwell and Riordan 1991), signaling activity gives rise to overinvestment in costly actions. Spence (1974), Ireland (1994) and Andersson (1996) consider the welfare-maximizing tax on signals. In contrast, we consider the profit-maximizing tax, which "over-taxes" signaling and causes a downward distortion in allocation when receivers observe the tax scheme.

The paper is also closely related to the literature on screening. Screening models, such as Mussa and Rosen (1978) and Maskin and Riley (1984), typically assume that buyers derive only intrinsic value from the seller's product. Our model differs in that the product has also a signaling value such that a buyer's equilibrium payoff depends on the information that the product conveys. Rayo (2013) also considers the optimal monopolistic pricing to sell signals, assuming that receivers observe the seller's mechanism. Whereas we assume additive separability in receivers' responses (e.g., wages) and the buyer's type (e.g., ability) for the buyer's preference, Rayo's adopts a multiplicative structure and employs novel screening techniques to characterize which types should be pooled into the same level of signal. Our contribution is twofold. First, whereas Rayo assumes additive separability in intrinsic and signaling values for the buyer's preference, which leads to the same downward distortion as in the above classic screening models, we allow for a more general structure and show that signaling can mitigate the screening distortion (see Section 5). More important, our paper also studies the case where receivers cannot observe the seller's mechanism and compares this to the observed case and a variety of other benchmarks. This enables us to assess how price transparency affects the degree of signaling and welfare. Friedrichsen (2018) considers a two-dimensional model where buyers differ in both their taste for intrinsic value and their desire for signaling value, with the price being publicly observed. Similar to our paper, the author shows that monopoly can yield higher welfare than perfect competition by preventing wasteful signaling. Calzolari and Pavan (2006) study information disclosure in a sequential screening model. They show that the upstream principal leaves the agent more rents if she discloses information about the agent's type to the downstream principal. In our model, the seller leaves the buyer more rents if receivers can observe the buyer's type than otherwise.

The unobserved tuition case belongs to the class of signal-jamming models proposed by Fudenberg and Tirole (1986). For example, in Holmström (1999), the labor market cannot distinguish the impact of the worker's ability from that of his effort on output. Therefore, the worker works harder to improve the market's perception of his ability. In comparison, in this paper, the labor market cannot distinguish the impact of the worker's ability from that of tuition on the education level. Thus, the school has an incentive to secretly cut tuition, thereby improving the market's perception and stimulating demand for education.

Our paper contributes to the literature on intermediate price transparency. Inderst and Ottaviani (2012) shows how product providers compete via commissions paid to consumer advisers. Commissions bias advice, so an increase in a firm's commission reduces consumers' willingness to pay if they observe the commission. In our model, cheaper tuition reduces the signaling value of education; thus, tuition cuts are less effective at stimulating demand when employers observe tuition than they would be otherwise. In Janssen and Shelegia (2015), a manufacturer chooses a wholesale price, retailers choose retail prices, and consumers search for the best deal. They show that retailers are less sensitive to wholesale price changes when consumers do not observe the price than otherwise, as uninformed consumers are more likely to continue to search when the retail price increases. In our model, by contrast, the worker is more sensitive to tuition changes when employers do not observe the tuition scheme than otherwise, as uninformed employers will have better (worse) beliefs over the worker's ability if they observe a higher (lower) education level.

The impact of price transparency on product price and demand elasticity is also similar to that of price transparency in sequential bargaining problems such as Hörner and Vieille (2009). In particular, Kaya and Liu (2015) consider bargaining between a long-run buyer and a sequence of short-run sellers. They show that prices are higher when the sellers can observe preceding sellers' offers than otherwise. This is because with observable prices, a change in a seller's price exerts an informational externality (which is absent with unobservable prices) on the subsequent seller, inducing the latter to change price in the same direction. This in turn implies that demand is more elastic when prices are unobservable.

Lastly, our paper relates to the growing literature on information intermediaries initiated by Biglaiser (1993). For example, Lizzeri (1999) studies the design of certification system and shows that a monopolistic certifier may disclose no information about the agent and capture all the surplus. In Chan et al. (2007), schools design grading systems to place their students in the job market. They show that schools have incentives to inflate grades to improve the market's perception of their students. In contrast to these models, our model incorporates screening in addition to signaling, as the designer cannot observe the agent's type. Similarly, Zubrickas (2015) studies a school's optimal grading policy when the students' abilities are privately known and the job market has myopic beliefs about the school's grading policies.

More recently, Biglaiser and Li (2018) consider an expert market in which a seller privately chooses effort before going to a middleman who decides whether to buy the seller's good after receiving a signal about the good's quality. They show that the informational externality of the middleman can either increase or reduce the seller's effort. Our model differs in that the agent's private information is exogenous and transmitted through signaling.

Organization. The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 considers a two-type example to explain the key economic forces. Section 4 states the general results. Section 5 discusses the underlying intuitions and the implications of our results. Section 6 studies the equilibrium selection in the unobserved tuition case and explores some extensions. Section 7 concludes the paper. All proofs are in the Appendix.

2 Model

For expositional convenience, we present our model in conformity with the seminal work of Spence (1973). In Appendix E, we describe in a parallel manner how to apply this model to other important applications, such as conspicuous consumption and advertising.

Players and actions. There is a school (*seller*), a worker (*buyer*) and multiple identical and competing firms, also referred to as *the labor market* (*receiver*). At the beginning of the game, the school chooses a tuition scheme $T : \mathbb{R}_+ \to \mathbb{R}_+$, which specifies the tuition fee for each education level z. Then, the worker decides how much education to purchase from the school after observing the tuition scheme. For simplicity, we do not explicitly model firms' actions; rather, we directly assume that they offer the worker a wage equal to his expected productivity (see below).

The worker's productivity depends on both his ability (*type*) θ and education level z. Specifically, θ is a random variable, which is distributed over the interval $[\underline{\theta}, \bar{\theta}]$ according to a distribution function $F(\theta)$ with a positive density function $f(\theta)$. Let $Q(z, \theta)$ denote the productivity of a type-θ worker with education level z. We assume that $Q(z, \theta)$ is smooth, with Q_z, $Q_\theta > 0$ if $z > 0$, and $Q_{zz} \leq 0$. We also assume that education is essential in that a worker with no education has zero productivity irrespective of his ability, i.e., $Q(0, \theta) \equiv 0$. We consider this assumption realistic, as many jobs require a minimal education level. For example, a lawyer candidate must graduate from a law school, and medical school education is prerequisite for being a licensed practitioner of medicine. In Section 6, as extensions, we consider the case in which education is nonessential, i.e., $Q(0, \theta) \geq 0$ and $Q_\theta > 0$, and the case in which education is a pure signal, i.e., $Q_z \equiv 0$ for all θ.

Information. The worker's education level is publicly observed, but his ability is privately known. However, the prior distribution F is common knowledge. In this paper, we mainly study two variants of the model: in the *observed* case, the tuition scheme is observed by the labor market; in the *unobserved* case, it is unobserved by the labor market. In each case, given the available information, the labor market chooses a wage schedule $W : \mathbb{R}_+ \to \mathbb{R}_+$, which specifies the wage for each education level z.

Payoffs. We normalize the school's cost of providing education to zero. Suppose the school chooses some tuition scheme T; then, $z(\theta; T)$ denotes the education level chosen by a type-θ worker under T. Given the tuition scheme T and a wage schedule W, a type-θ worker who chooses education level z has utility

$$u(z, \theta) := W(z) - C(z, \theta) - T(z),$$

where $C(z, \theta)$ denotes the worker's effort cost of education. Assume that $C(z, \theta)$ is smooth, with $C_z > 0$ if $z > 0$ and $C_{zz} > k$ for some $k > 0$. Moreover, the standard *single-crossing property* holds: $C_{z\theta} < 0$. This condition reflects the feature that a higher-ability worker has lower marginal effort costs than a lower-ability worker. We further normalize $C(0, \theta)$ to 0 for all $\theta \in [\underline{\theta}, \bar{\theta}]$. This implies that, combined with $C_{z\theta} < 0$, $C_\theta < 0$ if and only if $z > 0$. Finally, we assume that the worker can obtain a zero-utility outside option by acquiring no education and not entering the labor market.

First-best benchmark. Define $S(z, \theta)$ as the social surplus function, i.e.,

$$S(z, \theta) := Q(z, \theta) - C(z, \theta).$$

Note that $S(z, \theta)$ is strictly concave in z and thus has a unique maximizer $z^{fb}(\theta)$. Assume that $z^{fb}(\theta) \geq 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$, with equality possibly holding at $\underline{\theta}$ only. Thus, $z^{fb}(\theta)$ is determined by the first-order condition

$$S_z(z^{fb}(\theta), \theta) = Q_z(z^{fb}(\theta), \theta) - C_z(z^{fb}(\theta), \theta) = 0. \tag{2.1}$$

By the maximum theorem, $z^{fb}(\theta)$ is continuous. We also assume that $S(z, \theta)$ has increasing differences in both arguments: $S_{z\theta} > 0$. This means that a higher-ability worker can generate higher marginal surplus through education. It follows that $z^{fb}(\theta)$ is increasing on $[\underline{\theta}, \bar{\theta}]$.

Equilibrium. We use *perfect Bayesian equilibrium* as the solution concept throughout the paper. In the observed case, an equilibrium is a tuition scheme T^o, and conditional on each tuition scheme T, an education function $z^o(\theta; T)$, a wage schedule $W^o(z; T)$, and the labor market's posterior belief about the worker's ability, or simply *the market belief*, such that

(i) For each T: given $W^o(z;T)$, $z^o(\theta;T)$ maximizes $u(z,\theta)$; $W^o(z;T) = \mathbb{E}[Q(z,\theta)|z^o(\theta;T)]$ with the market belief updated by Bayes' rule whenever possible.

(ii) T^o maximizes the school's expected profit $\mathbb{E}[T(z^o(\theta;T))]$ among all T.

In the unobserved case, the market's inference is independent of the actual tuition scheme but is conditional on a *conjectured* tuition scheme; in equilibrium, the conjecture is correct. Formally, an equilibrium is a tuition scheme T^u, a wage schedule W^u along with the market belief, and conditional on each tuition scheme T, an education function $z^u(\theta;T)$, such that

(i) Given W^u, for each T, $z^u(\theta;T)$ maximizes $u(z,\theta)$; $W^u(z) = \mathbb{E}[Q(z,\theta)|z^u(\theta;T^u)]$ with the market belief updated by Bayes' rule whenever possible.

(ii) T^u maximizes the school's expected profit $\mathbb{E}[T(z^u(\theta;T))]$ among all T.

As in standard signaling games, there may exist multiple equilibria. To this end, in the observed case we focus on the *seller-optimal equilibrium*, i.e., the equilibrium that yields the highest payoff for the seller. This is essentially allowing the school to communicate with the worker and the labor market, implicitly through the tuition scheme, as to which signaling equilibrium to play in each subgame. In the unobserved case, we propose a novel refinement, *quasi-divinity*, which uniquely selects the *seller-optimal separating equilibrium*, i.e., the most profitable equilibrium for the seller provided that $z(\theta)$ is one-to-one if $z > 0$. In Section 6, we will discuss the equilibrium selection in the unobserved case in greater detail.

2.1 Direct mechanisms

Appealing to the revelation principle, we now focus on direct mechanisms between the school and the worker in both the observed and unobserved cases. Specifically, the school offers the worker a contract $\{z(\theta), T(\theta)\}$, and both players know which wage schedule the labor market will choose (as prescribed by equilibrium). Then, the worker privately reports a type to the school; thus, the labor market will observe the worker's education level but not his report.[2] Finally, the labor market chooses a wage schedule $W(z)$ based on the available information: in the observed case, the labor market observes the contract; in the unobserved case, it does not. Reporting a type $\hat{\theta}$, the worker obtains education $z(\hat{\theta})$, pays tuition $T(\hat{\theta})$, and receives wage $W(z(\hat{\theta}))$. Hence, a direct mechanism (or simply *mechanism*) consists of an allocation rule $z(\theta)$ and a transfer rule $T(\theta) - W(z(\theta))$, as in classic mechanism design problems.

[2] If reports are public, then the set of outcomes that can be implemented by a truthful direct mechanism is smaller than what can be obtained by an indirect mechanism in which the worker chooses education.

Worker's problem. In both cases, given a mechanism, a type-θ worker chooses a report $\hat{\theta}$ to maximize his utility

$$u(\hat{\theta}, \theta) \coloneqq W(z(\hat{\theta})) - C(z(\hat{\theta}), \theta) - T(\hat{\theta}).$$

A mechanism is *incentive compatible* if the worker is willing to truthfully report his type, and is *individually rational* if the worker obtains nonnegative utility. Let $U(\theta) \coloneqq u(\theta, \theta)$ be a type-θ worker's equilibrium utility. We say that a mechanism is *implementable* if it satisfies incentive compatibility (IC) and individual rationality (IR). Appealing to Mas-Colell et al. (1995, Proposition 23.D.2), we characterize all implementable mechanisms below.

Lemma 1. *In both cases, a mechanism is implementable if and only if*

(i) $z(\theta)$ *is nondecreasing.*

(ii) Define $\theta_0 \coloneqq \inf\{\theta | z(\theta) > 0\}$. *For any* $\theta < \theta_0$, $U(\theta) = 0$*; for any* $\theta \geq \theta_0$,

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} -C_\theta(z(s), s) ds \geq 0.$$

School's problem. In the observed case, since we focus on the seller-optimal equilibrium, the school's problem can be formulated as choosing a mechanism to maximize its expected profit subject to the IC and IR constraints and the market belief being correct. In contrast, in the unobserved case, since the market's inference is independent of the school's choice, given a wage schedule, the school chooses a contract, rather than a mechanism, to maximize its expected profit subject to the IC and IR constraints.

From Lemma 1, we can rewrite the school's problem for both cases. Note that IC means that $T(\theta) = W(z(\theta)) - C(z(\theta), \theta) - U(\theta)$ and that $U(\theta_0)$ is optimally set to 0. Substituting and integrating by parts, the school's problem in both cases can be formulated as choosing an allocation rule $z(\theta)$ to maximize its expected profit

$$\int_{\theta_0}^{\bar{\theta}} \left[W(z(\theta)) - C(z(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z(\theta), \theta) \right] dF(\theta) \tag{2.2}$$

subject to $z(\theta)$ being nondecreasing. Note that the terms between brackets amount to the *virtual surplus* in mechanism design theory, except that the wage $W(z)$ is now endogenous. It is helpful to define the sum of the effort cost and consumer surplus (rent) as

$$G(z, \theta) \coloneqq C(z, \theta) + \frac{1 - F(\theta)}{f(\theta)} \left[-C_\theta(z, \theta) \right].$$

Then, the virtual surplus is simplified as $W(z) - G(z, \theta)$. Since $C_{z\theta} < 0$, we have $G_z \geq C_z$ for all $z \in \mathbb{R}_+$, with equality holding at $\bar{\theta}$ only.

Equilibrium. In the observed case, market belief correctness means that for each allocation rule $z(\theta)$, $W(z) = \mathbb{E}[Q(z,\theta)|z(\theta)]$. While $W(z(\theta))$ does not necessarily equal $Q(z(\theta),\theta)$ when $z(\theta)$ is constant, the law of total expectation implies that program (2.2) is equivalent to[3]

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} [Q(z(\theta),\theta) - G(z(\theta),\theta)]\, dF(\theta) \tag{2.3}$$

subject to $z(\theta)$ being nondecreasing. Thus, to characterize the equilibrium of the observed case, it suffices to solve program (2.3). Let $z^o(\theta)$ be an optimal allocation rule with cutoff type θ_0^o. Let T^o and W^o be the associated tuition scheme and wage schedule, respectively.

In the unobserved case, an equilibrium is an allocation rule $z^u(\theta)$ and a wage schedule W^u along with the market belief, such that (i) given W^u, $z^u(\theta)$ solves program (2.2); (ii) $W^u(z) = \mathbb{E}[Q(z,\theta)|z^u(\theta)]$ with the market belief updated by Bayes' rule whenever possible. Let T^u and θ_0^u be the associated tuition scheme and the cutoff type, respectively. In case of multiple equilibria, we focus on the seller-optimal separating equilibrium.

To guarantee that an equilibrium exists in both cases, we assume that $G_{zz} > k$ for some $k > 0$, which is ensured by $C_{zz\theta} \leq 0$. Moreover, we assume that $G_{z\theta} < 0$, i.e., it is less costly for the school to serve a higher-ability worker. That is, we impose a single-crossing property on the virtual surplus. This is ensured by $C_{z\theta\theta} \geq 0$ and the assumption below.

Assumption 1. *$d[(1 - F(\theta))/f(\theta)]/d\theta < 1$.*

Assumption 1 is clearly less restrictive than the standard monotone hazard rate property in mechanism design theory, since it only requires the slope of the hazard rate to not be too negative. Throughout, we assume that $C_{zz\theta} \leq 0$, $C_{z\theta\theta} \geq 0$ and Assumption 1 holds.

2.2 Spencian job market signaling

As a reference point, we briefly revisit Spence's signaling game in which tuition is fixed at zero, as if competitive schools set tuition at the marginal cost. In this case, an equilibrium is an education function $z^s(\theta)$ and a wage schedule W^s along with the market belief, such that (i) given W^s, $z^s(\theta)$ maximizes $u(z,\theta)$; (ii) $W^s(z) = \mathbb{E}[Q(z,\theta)|z^s(\theta)]$ with the market belief updated by Bayes' rule whenever possible. Following the signaling literature, we focus on the least-cost separating equilibrium (i.e., the Riley outcome). Appealing to Mailath and

[3] In Rayo (2013), the buyer's preference has a multiplicative structure such that his marginal utility from the receiver's action is increasing in his type, whereas in our model with additive separability, such marginal utility is independent of the buyer's type. Thus, in contrast to our model, the law of total expectation cannot simplify the seller's problem in Rayo's model, in which a novel screening technique is provided.

von Thadden (2013, Theorems 2, 5 and 6), we have that such an equilibrium exists, in which $z^s(\theta)$ is continuous and increasing on $[\underline{\theta}, \bar{\theta}]$ and satisfies the differential equation

$$Q_z(z^s(\theta), \theta) + Q_\theta(z^s(\theta), \theta)\theta^{s\prime}(z^s(\theta)) - C_z(z^s(\theta), \theta) = 0 \tag{2.4}$$

on $(\underline{\theta}, \bar{\theta}]$ with $z^s(\underline{\theta}) = z^{fb}(\underline{\theta})$, where $\theta^s(z)$ is the inverse of $z^s(\theta)$, which is also differentiable on $(\underline{\theta}, \bar{\theta}]$. Since $z^s(\theta)$ is increasing, we have $W^s(z^s(\theta)) = Q(z^s(\theta), \theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

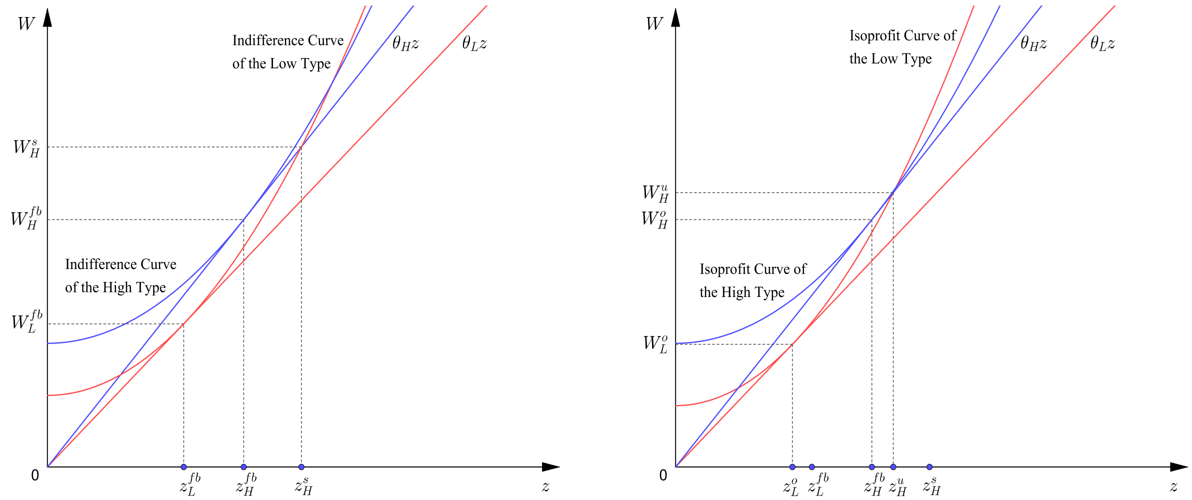
Note that the first two terms on the left-hand side (LHS) of (2.4) are the total derivative of $W^s(z^s(\theta))$. In particular, the second term is nonnegative given the monotonicity of $z^s(\theta)$. Since $S(z, \theta)$ is strictly concave in z, comparing (2.4) with (2.1) implies that $z^s(\theta) \geq z^{fb}(\theta)$ for all $\theta \geq \underline{\theta}$, with equality holding at $\underline{\theta}$ only. This result means that in Spence's game, the worker chooses more education than the first best; that is, signaling leads to over-education. The intuition is well known. Under complete information, the marginal benefit of education is its marginal contribution to human capital. In contrast, when ability is privately known, in addition to the human capital effect, there is a *signaling effect*; that is, a higher education level makes the labor market regard the worker as having higher ability. Thus, the marginal benefit of education is higher than under complete information.

3 A two-type example

To provide simple intuitions for our main results, we consider a two-type example. Assume that $\theta \in \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H$ and that both types are equally likely. Assume further that $Q(z, \theta) = \theta z$ and $C(z, \theta) = z^2/(2\theta)$. The analysis can be developed graphically.

In Panel (a) of Figure 1, we depict each type's indifference curve and productivity. The first-best allocations z_L^{fb} and z_H^{fb} are determined by the tangent points between each type's indifference curve and productivity line. Note that the low type θ_L strictly prefers (z_H^{fb}, W_H^{fb}) to (z_L^{fb}, W_L^{fb}). Therefore, the first-best outcome cannot be an equilibrium of Spence's game, as it violates the incentive constraint: $W_L - C(z_L, \theta_L) \geq W_H - C(z_H, \theta_L)$. In the least-cost separating equilibrium, the low type chooses the first-best education level z_L^{fb}, whereas the high type chooses a higher level z_H^s than the first best z_H^{fb} to separate himself from the low type, so that the incentive constraint binds (i.e., the low type's indifference curve intersects with the high type's productivity line). That is, signaling induces over-education.

In Panel (b) of Figure 1, we start to consider the school's strategic choice. Analogously, we depict each type's isoprofit curve that yields a constant virtual surplus $W - G(z, \theta)$, and his productivity line. Note that the low type's isoprofit curve is steeper than his indifference



(a) Tuition equals zero

(b) School chooses tuition

Figure 1

curve, whereas for the high type, the two curves coincide.[4] In the observed case, market belief correctness implies that $W_L + W_H = \theta_L z_L + \theta_H z_H$ irrespective of whether $z_L = z_H$. Thus, the seller-optimal equilibrium is given analogously by the tangent point between each type's isoprofit curve and productivity line, in which the low type chooses a lower education level z_L^o than the first best z_L^{fb}, whereas the high type chooses exactly the first best z_H^{fb}. That is, in stark contrast to Spence's game, the observed case exhibits under-education. The altered equilibrium results from screening. Intuitively, with lower effort costs, the high type has an incentive to mimic the low type. To incentivize truth-telling, the school leaves the high type an information rent $C(z_L, \theta_L) - C(z_L, \theta_H)$. Since $C_{z\theta} < 0$, we have $Q_z(z, \theta_L) - G_z(z, \theta_L) < Q_z(z, \theta_L) - C_z(z, \theta_L) = S_z(z, \theta_L)$. That is, the marginal virtual surplus (marginal profit) of the low type is less than the marginal social surplus. This induces the school to undersupply education to the low type. Since there is no higher type to mimic the high type, he receives the first-best education level; that is, there is no distortion at the top.

Now, let us consider the unobserved case. Suppose the labor market believes naively that the school's contract is the same as that in the observed case and thus offers the same wage schedule. Will the school choose the same contract? The answer is no. To see this, consider an alternative contract such that the school only offers the high education level z_H^{fb} from the observed case and reduces tuition to the level that also attracts the low type. This leads to a profitable deviation because the point (z_H^{fb}, W_H^o) is strictly above the isoprofit curve of the

[4] According to Section 2.1, we have $G(z, \theta_L) = C(z, \theta_L) + [C(z, \theta_L) - C(z, \theta_H)]$ and $G(z, \theta_H) = C(z, \theta_H)$. Since $C_{z\theta} < 0$, we have $G_z(z, \theta_L) > C_z(z, \theta_L)$. Thus, the statement is proven.

low type, as depicted in Panel (b). Intuitively, since the market belief is the same as in the observed case, the school has an incentive to induce the low type to mimic the high type via secret price cuts, thereby collecting a higher revenue from the worker. Thus, in equilibrium, the high education level must be so high that the school finds it too costly to induce the low type to deviate, i.e., the incentive constraint must hold: $W_L - G(z_L, \theta_L) \geq W_H - G(z_H, \theta_L)$. In the seller-optimal separating equilibrium, the low type chooses the same education level as in the observed case, whereas the high type chooses a higher level than that of the observed case (which is the first-best), so that the above incentive constraint binds (i.e., the low type's isoprofit curve intersects with the high type's productivity line). That is, the worker obtains more education in the unobserved case. Since in both cases the low type's allocation is the same and the high type's IC constraint binds, each type gains the same utility in both cases. However, in the unobserved case the high education level z_H^u is inefficient; thus, to make the high type no worse off, the price of the high education level must be lower. This implies that both tuition and the school's payoff are lower in the unobserved case. Lastly, note that both types choose less education in the unobserved case than in Spence's game. This is because the low type's isoprofit curve is steeper than his indifference curve, thereby tangent to the low productivity line and intersecting with the high productivity line both at lower points.

In summary, this example illustrates that in the observed case, the worker chooses less education than the first-best level. In the unobserved case, he chooses more education than in the observed case but less than in Spence's game. Moreover, in the unobserved case, both tuition and the school's payoff are lower, but the worker's utility is (weakly) higher than in the observed case. In the next section, we generalize these qualitative results.

4 The results: The role of price transparency

In this section, we state the general results of the paper.

4.1 Labor market observes tuition

We first consider the observed case. From Section 2.1, we can characterize the seller-optimal equilibrium by solving program (2.3). Define the virtual surplus in the observed case as

$$J^o(z, \theta) := Q(z, \theta) - G(z, \theta).$$

Note that $J^o(z, \theta)$ is strictly concave in z and thus has a unique maximizer on $\mathbb{R}+$, denoted $z^*(\theta)$. Moreover, we say that the virtual surplus in the observed case is *regular* if $J^o(z, \theta)$

also has increasing differences in z and θ. Whereas the model assumptions only imply that $G_{z\theta} < 0$, it is immediate that if $Q_{z\theta} \geq 0$ in addition, then $J^o(z, \theta)$ is regular. It follows that in this case $z^*(\theta)$ is continuous and increasing on $[\underline{\theta}, \bar{\theta}]$. Thus, the school's problem reduces to pointwise maximization for $J^o(z, \theta)$ under regularity.

In contrast, when regularity does not hold, $z^*(\theta)$ might be decreasing in some region. In this case, we adopt the generalized ironing technique developed by Toikka (2011). Define

$$H(z, \theta) \coloneqq \int_{\underline{\theta}}^{\theta} J_z^o(z, s) ds.$$

Since $J_z^o(z, \theta)$ is continuous, $H(z, \theta)$ is continuously differentiable on $[\underline{\theta}, \bar{\theta}]$ for any fixed z. Let $I(z, \cdot) \coloneqq \text{conv } H(z, \cdot)$ be the convex hull of $H(z, \cdot)$; thus, $I(z, \cdot)$ is continuously differentiable on $[\underline{\theta}, \bar{\theta}]$, and $I_\theta(z, \theta)$ is nondecreasing in θ. Define the *generalized virtual surplus* as

$$\bar{J}^o(z, \theta) \coloneqq J^o(0, \theta) + \int_0^z I_\theta(x, \theta) dx.$$

Our first theorem characterizes the equilibrium outcome of the observed case.

Theorem 1. *In the observed case, the seller-optimal equilibrium exists such that*

$$z^o(\theta) = \begin{cases} \bar{z}(\theta) & \text{if } \theta \geq \theta_0^o \\ 0 & \text{otherwise,} \end{cases} \tag{4.1}$$

where $\bar{z}(\theta)$ is the unique maximizer of $\bar{J}^o(z, \theta)$ and the cutoff type θ_0^o is either the maximal root of $\bar{z}(\theta) = 0$ if it exists or $\underline{\theta}$ otherwise. $z^o(\theta)$ is nondecreasing and continuous on $[\underline{\theta}, \bar{\theta}]$. If $z^*(\underline{\theta}) > 0$, then $\theta_0^o = \underline{\theta}$. If $\theta_0^o > \underline{\theta}$, then $z^o(\theta_0^o) = z^*(\theta_0^o) = 0$. In particular, when $J^o(z, \theta)$ is regular, $z^o(\theta) \equiv z^*(\theta)$ on $[\underline{\theta}, \bar{\theta}]$. Then, for each $z^o(\theta)$ with $\theta \in [\theta_0^o, \bar{\theta}]$,[5]

$$T^o(z^o(\theta)) = W^o(z^o(\theta)) - C(z^o(\theta), \theta) + \int_{\theta_0^o}^{\theta} C_\theta(z^o(s), s) ds, \tag{4.2}$$

where $W^o(z^o(\theta)) = \mathbb{E}[Q(z^o(\theta), \theta)]$ with the market belief updated by Bayes' rule.

Theorem 1 states that the optimal allocation rule $z^o(\theta)$ is continuous and coincides with the unconstrained optimizer $z^*(\theta)$ under the regularity of $J^o(z, \theta)$. In the proof, we also show that whenever $z^o(\theta) \neq z^*(\theta)$, θ belongs to a *pooling* interval (i.e., an ironing region), so that $z^o(\theta)$ is constant in this interval. It thus follows from the continuity of $z^o(\theta)$ that $W^o(z)$ is discontinuous at such $z^o(\theta)$. Then, by (4.2), $T^o(z)$ is also discontinuous at such $z^o(\theta)$.

[5] For any off-path $z > 0$, it is conventional to simply assume that $T^o(z) = +\infty$.

Since $G_z \geq C_z$, we have $J_z^o(z,\theta) \leq S_z(z,\theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$, with equality holding at $\bar{\theta}$ only. This states, as established in the two-type example, that the worker's isoprofit curve is steeper than his indifference curve due to the information rent. Then, the next proposition generalizes the previous result that $z^o(\theta)$ exhibits downward distortion.

Proposition 1. *In the observed case, the worker acquires less education than the first best. Specifically,* $z^o(\theta) \leq z^{fb}(\theta)$ *on* $[\underline{\theta}, \bar{\theta}]$, *with strict inequality on* $(\underline{\theta}, \bar{\theta})$ *and* $z^o(\bar{\theta}) = z^{fb}(\bar{\theta})$.

4.2 Labor market does not observe tuition

We now turn to the unobserved case and focus on the seller-optimal separating equilibrium. Similarly, define the virtual surplus in the unobserved case as

$$J^u(z,\theta) \coloneqq W(z) - G(z,\theta).$$

The next theorem states that the seller-optimal separating equilibrium always exists.

Theorem 2. *The seller-optimal separating equilibrium always exists, in which* $\theta_0^u = \theta_0^o$, *and* $z^u(\theta)$ *is continuous and increasing on* $[\theta_0^o, \bar{\theta}]$ *and satisfies the differential equation*

$$Q_z(z^u(\theta),\theta) + Q_\theta(z^u(\theta),\theta)\theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta),\theta) = 0 \tag{4.3}$$

on $(\theta_0^o, \bar{\theta}]$ *with* $z^u(\theta_0^o) = z^*(\theta_0^o)$, *where* $\theta^u(z)$ *is the inverse of* $z^u(\theta)$, *which is also differentiable on* $(\theta_0^o, \bar{\theta}]$. *Then, for each* $z^u(\theta)$ *with* $\theta \in [\theta_0^o, \bar{\theta}]$, $W^u(z^u(\theta)) = Q(z^u(\theta),\theta)$ *and*

$$T^u(z^u(\theta)) = W^u(z^u(\theta)) - C(z^u(\theta),\theta) + \int_{\theta_0^o}^{\theta} C_\theta(z^u(s),s)ds. \tag{4.4}$$

Theorem 2 indicates that the unobserved case shares the same cutoff type (i.e., $\theta_0^u = \theta_0^o$) and thus the same market coverage with the observed case. It also states that type θ_0^o chooses the unconstrained optimal education level $z^*(\theta_0^o)$. This initial condition uniquely pins down the equilibrium education function $z^u(\theta)$ and thus the equilibrium outcome.

Our third theorem presents the paper's main result. In contrast with the observed case, the worker acquires more education in the unobserved case. In particular, each inframarginal type chooses strictly more education than in the observed case, as illustrated in Figure 2.

Theorem 3. *In contrast with the observed case, the worker acquires more education in the unobserved case. Specifically,* $z^u(\theta) \geq z^o(\theta)$ *on* $[\underline{\theta}, \bar{\theta}]$, *with strict inequality for* $\theta > \theta_0^o$.

As an immediate result of Theorem 3, the school's equilibrium payoff in the unobserved case, denoted Π^u, is lower than that in the observed case, denoted Π^o. Formally, we have

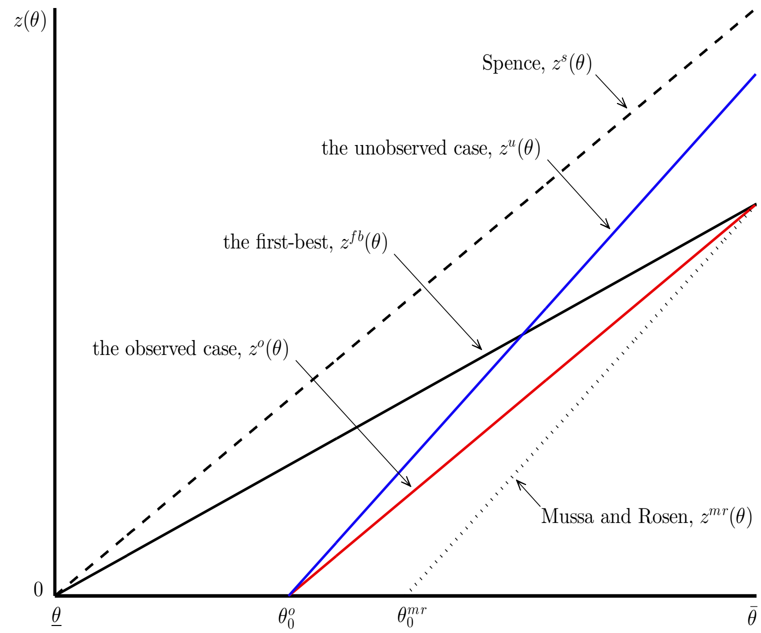


Figure 2. This figure illustrates all the equilibrium education functions considered in this paper. The figure assumes that $Q(z, \theta) = \theta z + z$, $C(z, \theta) = z^2 + z - \theta z$, and $\theta \sim U[0, 1]$. As a result, $z^{fb}(\theta) = \theta$, $z^s(\theta) = 3\theta/2$, $z^o(\theta) = (3\theta - 1)/2$, $z^u(\theta) = 2\theta - 2/3$, and $z^{mr}(\theta) = 2\theta - 1$.

Corollary 1. *In contrast with the observed case, the school obtains a strictly lower expected profit in the unobserved case. That is,* $\Pi^u < \Pi^o$.

In terms of the worker's payoff, note that the worker gains more information rents since he obtains more education. Formally, let $U^o(\theta)$ and $U^u(\theta)$ be type θ's equilibrium utility in the observed and unobserved cases, respectively. By Theorem 3, for any $\theta \in (\theta_0^o, \bar{\theta}]$,

$$U^u(\theta) - U^o(\theta) = \int_{\theta_0^o}^{\theta} [C_\theta(z^o(s), s) - C_\theta(z^u(s), s)] ds > 0.$$

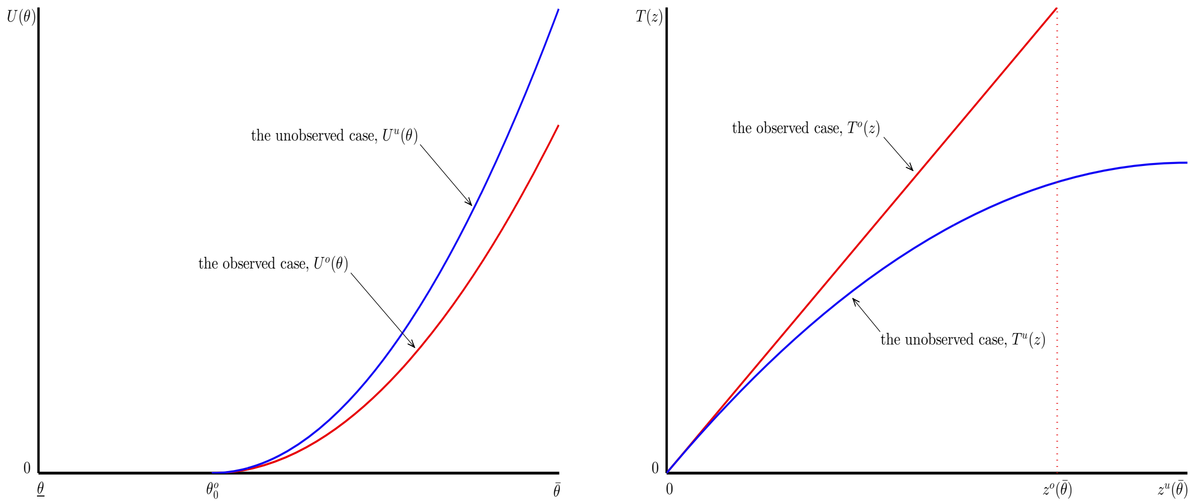
That is, the worker has higher utility in the unobserved case than in the observed case (see Figure 3). This generalizes the knife-edge result of the two-type example. To summarize,

Corollary 2. $U^u(\theta) \geq U^o(\theta)$ *on* $[\underline{\theta}, \bar{\theta}]$*, with strict inequality for* $\theta > \theta_0^o$.

Then, we show that education is more expensive in the observed case. Specifically, the tuition scheme in the unobserved case is uniformly lower than that in the observed case on the common interval of education, i.e., $[z^*(\theta_0^o), z^*(\bar{\theta})]$ (see Figure 3). Formally, we have

Proposition 2. $T^o(z) \geq T^u(z)$ *on* $[z^*(\theta_0^o), z^*(\bar{\theta})]$*, with strict inequality for* $z > z^*(\theta_0^o)$.

Finally, we generalize the result that the equilibrium education levels of the unobserved case are bounded above by that of Spence's game (see Figure 2). Formally, we have



(a) Worker's utility

(b) Tuition scheme

Figure 3. This figure compares the worker's utility and tuition between the observed and unobserved cases. The figure considers the same example as Figure 2. As a result, (a) $U^o(\theta) = \frac{3}{4}(\theta - \frac{1}{3})^2$ and $U^u(\theta) = (\theta - \frac{1}{3})^2$; (b) $T^o(z) = \frac{2z}{3}$ and $T^u(z) = -\frac{z^2}{4} + \frac{2z}{3}$.

Proposition 3. *In the unobserved case, the worker acquires less education than in Spence's signaling game. Specifically, $z^u(\theta) \leq z^s(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \underline{\theta}$.*

The idea of Proposition 3 is simple. Note that the unobserved case is essentially Spence's game with higher marginal effort costs (i.e., $G_z \geq C_z$); thus, the worker's "indifference curve" is steeper in the unobserved case than in Spence's game, leading to lower education levels.

5 The economics of optimal pricing

To see the intuitions of the general results, it is instructive to investigate the optimal tuition of each variant of our model. As a benchmark, we have shown that if tuition is fixed at the marginal cost, then signaling will induce over-education. To restore the first-best outcome, a social planner would levy Pigouvian taxes to undo the signaling effect (e.g., Spence (1974)). Let T^{fb} denote the welfare-maximizing tax on education. The marginal tax is equal to the signaling effect at the first best, i.e.,

$$T^{fb\prime}(z) = Q_\theta(z, \theta^{fb}(z))\theta^{fb\prime}(z), \tag{5.1}$$

where $\theta^{fb}(z)$ is the inverse of $z^{fb}(\theta)$, which is differentiable on $[\underline{\theta}, \bar{\theta}]$.

Then, we consider the profit-maximizing school's pricing strategy. In the observed case, recall that the school undersupplies education due to the mechanism of monopoly screening.

However, this outcome results from the interaction between screening and signaling. To see the role of signaling, note that given T^o, the subgame is essentially Spence's game as if the worker had a cost function given by $C(z,\theta)+T^o(z)$. It follows from the same argument as in Section 2.2 that the worker overinvests in education in terms of $C(z,\theta)+T^o(z)$.[6]

However, Proposition 1 states that when both screening and signaling exist and exert the opposite effects—screening induces under-education, but signaling induces over-education—screening outweighs signaling. This is because as a Stackelberg leader, the school internalizes the worker's signaling activity when screening his type. To illustrate, assume for simplicity that $z^o(\theta)$ is increasing and, thus, $T^o(z)$ is differentiable. Then, by the first-order condition of the worker's optimal choice, for each $z \in [z^o(\theta_0^o), z^o(\bar{\theta})]$, we have

$$T^{o\prime}(z) = W^{o\prime}(z) - C_z(z,\theta^o(z)) = \frac{d}{dz}\left[Q(z,\theta^o(z))\right] - C_z(z,\theta^o(z)),$$

where $\theta^o(z)$ is the inverse of $z^o(\theta)$, which is differential on $[\theta_0^o, \bar{\theta}]$. Substituting this equation into the first-order condition of $J^o(z,\theta)$ with respect to z, we have

$$T^{o\prime}(z) = Q_\theta(z,\theta^o(z))\theta^{o\prime}(z) + \frac{1-F(\theta^o(z))}{f(\theta^o(z))}\left[-C_{z\theta}(z,\theta^o(z))\right]. \tag{5.2}$$

On the right-hand side (RHS) of (5.2), the first term is the signaling effect, and the second term is the marginal information rent extracted by the worker. Note that signaling induces over-education, which harms the school's profit in two ways: on the one hand, it reduces the social surplus; on the other hand, it provides the worker with more information rents. Thus, the optimal tuition scheme must undo these two effects, as indicated by (5.2). Because the second term on the RHS of (5.2) is positive, comparing (5.2) with (5.1) indicates that the profit-maximizing scheme "over-taxes" signaling and thus leads to under-education.

While screening outweighs signaling, signaling can in turn mitigate the distortion caused by screening. To see this, consider an otherwise identical model assuming that now the labor market also observes the worker's ability. Therefore, the wage equals the actual productivity, and signaling is eliminated. This means that the worker's reservation price for education is the social surplus. Since $S_{z\theta}>0$, a higher-ability worker derives higher marginal utility from education. Thus, the school has the same screening problem as in Mussa and Rosen (1978). Analogously, the virtual surplus in Mussa and Rosen's game is given by

$$J^{mr}(z,\theta) := S(z,\theta) - \frac{1-F(\theta)}{f(\theta)}S_\theta(z,\theta).$$

[6]To illustrate, consider the example in Figure 2. In terms of the total cost $C(z,\theta)+T^o(z)$, the first-best education level is $z^{fb}(\theta)=\theta-\frac{1}{3}$, while $z^o(\theta)=\frac{3\theta-1}{2}$. Thus, $z^o(\theta)\geq z^{fb}(\theta)$ with strict inequality on $(\frac{1}{3},1]$.

Let $z^{mr}(\theta)$ and θ_0^{mr} be an optimal allocation rule and the associated cutoff type, respectively. For simplicity, assume that both $J^o(z,\theta)$ and $J^{mr}(z,\theta)$ are regular.[7] Thus, $z^{mr}(\theta)$ and θ_0^{mr} can be solved by pointwise maximization for $J^{mr}(z,\theta)$.

Then, we illustrate how the allocation in Mussa and Rosen's game is different from that in the observed case. On the extensive margin, since $S_\theta > -C_\theta$ if $z > 0$, $J^{mr}(z,\theta) \leq J^o(z,\theta)$, holding weakly on the boundary. Therefore, if $\theta_0^o > \underline{\theta}$, then $\theta_0^{mr} > \theta_0^o$; that is, more types will be excluded in Mussa and Rosen's game. On the intensive margin, if $Q_{z\theta} > 0$ on $[0, z^{fb}(\bar{\theta})]$,[8] then $z^{mr}(\theta) \leq z^o(\theta)$ with strict inequality on $[\theta_0^o, \bar{\theta})$, meaning that under-education is more significant in Mussa and Rosen's game. These findings are illustrated in Figure 2.

For welfare comparison, recall that education is already undersupplied in the observed case, yet the downward distortion is larger in Mussa and Rosen's game; thus, the observed case yields higher social welfare. In addition, since $J^{mr}(z^{mr}(\theta),\theta) \leq J^o(z^o(\theta),\theta)$ with strict inequality on $[\theta_0^o, \bar{\theta})$, and $\theta_0^{mr} \geq \theta_0^o$, it is readily confirmed that the school's expected profit is also higher in the observed case. In summary, we have the following proposition.

Proposition 4. *Suppose both $J^o(z,\theta)$ and $J^{mr}(z,\theta)$ are regular and $Q_{z\theta} > 0$ on $[0, z^{fb}(\bar{\theta})]$, then under-education will be greater if signaling is eliminated. Specifically, $z^{mr}(\theta) \leq z^o(\theta)$, with strict inequality on $[\theta_0^o, \bar{\theta})$. If $\theta_0^o > \underline{\theta}$, then $\theta_0^{mr} > \theta_0^o > \underline{\theta}$. Moreover, the school's expected profit and welfare are strictly higher when signaling is present than otherwise.*

Intuitively, when the labor market observes the worker's ability, if a higher type mimics a lower type by choosing the same education, he not only incurs a lower total cost than the latter but also receives a higher wage due to his higher productivity. In contrast, when the labor market does not observe the worker's ability, the higher type can no longer directly reap the benefit from higher productivity. Therefore, he acquires more education to signal his ability. The signaling incentive dampens the worker's temptation to mimic lower types. Thus, the school provides lower information rents to the worker when signaling is present. Formally, we have that for all $\theta \in [\underline{\theta}, \bar{\theta}]$ and $z > 0$,

$$\underbrace{\frac{1-F(\theta)}{f(\theta)}[-C_\theta(z,\theta)]}_{\text{information rents with signaling}} \leq \underbrace{\frac{1-F(\theta)}{f(\theta)}S_\theta(z,\theta)}_{\text{information rents without signaling}}$$

which holds with equality at $\bar{\theta}$ only. That is, signaling mitigates the screening distortion.[9]

[7]Given the model assumptions, $J^{mr}(z,\theta)$ is regular if $Q_{z\theta\theta} \leq 0$.

[8]This condition is not restrictive; indeed, given that $Q_\theta > 0$ if $z > 0$ and $Q(0,\theta) \equiv 0$, we have $Q_{z\theta} > 0$ for $z \in [0,k]$ for some $k > 0$. Given this condition, $J_z^{mr}(z,\theta) < J_z^o(z,\theta)$ on $[0, z^{fb}(\bar{\theta})]$ for $\theta < \bar{\theta}$.

[9]Rayo (2013) also considers the case where the signaling good provides intrinsic quality to the buyer in

Recall that in Spence's game, signaling reduces social welfare, as it causes over-education. In the observed case, by contrast, signaling raises social welfare, relative to Mussa and Rosen, since it mitigates the screening distortion. Hence, any instrument that attenuates signaling is socially beneficial in the Spencian world but harmful in the observed case. For example, students' grades substitute for their education levels in signaling. Suppose grades become less informative, e.g., due to grade inflation—an increasingly common phenomenon at American colleges and universities,[10]—then signaling through education will be enhanced, as students will attempt to separate themselves from others (Daley and Green, 2014). This implies that coarse grading can be socially beneficial in the observed case by relieving under-education,[11] whereas it is harmful in the Spencian world because it aggravates over-education.

Now, we turn to the unobserved case. Theorem 3 indicates that the worker obtains more education in the unobserved case than in the observed case. This result is driven by a signal-jamming effect: the school jams the worker's signal when the labor market does not observe the actual cost of education. To see the intuition, suppose the school lowers tuition so that the worker chooses more education than in the initial state. When the labor market observes the tuition change, it cuts wages, as any education level now corresponds to a lower-ability worker. In contrast, when the labor market does not observe the tuition change, it does not cut wages despite that tuition changes; thus, the worker is willing to pay more for additional education. Conversely, if the school raises tuition so that education decreases, then the labor market will raise wages in the observed case. As a result, the worker's willingness to pay will be lower in the unobserved case. This illustrates that the worker is more sensitive to tuition changes in the unobserved case than in the observed case.

That is, the school faces more elastic demand in the unobserved case. This induces the school to fool the labor market with secret price cuts. Specifically, the first-order condition of the worker's optimal choice means that $T^{u\prime}(z) = W^{u\prime}(z) - C_z(z, \theta^u(z))$. Substituting this equation into (4.3), and noting that $W^u(z) = Q(z, \theta^u(z))$, we have

$$T^{u\prime}(z) = \frac{1 - F(\theta^u(z))}{f(\theta^u(z))} \left[-C_{z\theta}(z, \theta^u(z)) \right]. \tag{5.3}$$

addition to signaling value, as in our model. A critical difference is that the buyer's preference in his model satisfies additive separability in quality and signaling value, and thus, the allocations of quality and signal interact only through the monotonicity constraint. It follows that the allocation of quality in Rayo's model coincides with that of Mussa and Rosen. That is, signaling cannot mitigate the screening distortion there.

[10] See, for example, Johnson (2006) and Rojstaczer and Healy (2010).

[11] Alternatively, Boleslavsky and Cotton (2015) shows that coarse grading can improve social welfare by enhancing schools' investments in education quality when schools compete in placing graduates.

Equation (5.3) states that in the unobserved case, the marginal tuition equals the marginal information rent extracted by the worker. In contrast to the observed case, as indicated by the comparison between (5.3) and (5.2), the optimal tuition scheme in the unobserved case does not undo the signaling effect. The reason is that the loss in total surplus caused by over-education will be compensated by the labor market overpaying the worker, as employers will overestimate the worker's ability if the school secretly cuts tuition. In equilibrium, the labor market correctly anticipates the school's incentive and offers lower wages, as education is inflated. This reduces the worker's willingness to pay, strictly for the inframarginal types; thus, the school gains lower profits. (5.3) also indicates that the marginal tuition vanishes at the highest education level, meaning that the school offers discounts (i.e., $T(z)/z$ is declining) for higher quantities, as in the classic screening model of Maskin and Riley (1984).

5.1 Policy implications

Our main results have meaningful implications for the price transparency of signaling goods. Proposition 2 and Corollary 2 mean that policies that improve the transparency of net prices at colleges and universities through mandatory disclosure may unintentionally induce more expensive education and harm students. These policies, such as U.S. Code § 1015a, require colleges to publicly disclose their net prices, which are usually not previously observed by employers. On the one hand, this reduces the search costs of students, thereby enhancing the competition between schools and lowering prices. On the other hand, this also allows schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers, financial aid and so forth. It is thus possible that such policies ultimately raise education costs and harm students. Hence, policymakers should not overlook the unintended negative effects of these mandatory disclosure policies.

Next, consider social welfare. On the one hand, Proposition 1 and Theorem 3 imply that $z^o(\theta) < z^u(\theta) < z^{fb}(\theta)$ in the right neighborhood of θ_0^o. That is, under-education is smaller at the left tail of the support in the unobserved case than in the observed case. Thus, welfare is higher in that region in the unobserved case. On the other hand, because $z^o(\bar{\theta}) = z^{fb}(\bar{\theta})$, $z^u(\theta) > z^{fb}(\theta)$ in the left neighborhood of $\bar{\theta}$; that is, there is over-education at the right tail of the support in the unobserved case. It follows from the continuity of $z^o(\theta)$ and $z^u(\theta)$ that welfare is lower in that region in the unobserved case. In general, whether the observed or unobserved case yields higher social welfare remains ambiguous.

Recall that in the observed case, signaling can reduce the screening distortion. Naturally, one might ask will signaling being more intense lead to more distortion cuts in the observed case but more overinvestment in the unobserved case? In Appendix D, we illustrate that if signaling is sufficiently intense (e.g., there is significant overinvestment in Spence's game), then the observed case yields higher social welfare than the unobserved case, and both cases yield higher social welfare than Spence's game. Among the three cases, as signaling intensity changes from low to high, the case that yields the highest social welfare will be Spence's game, the unobserved case, and the observed case, respectively.

This finding thus has welfare implications for the market structure of signals. When the market is served by perfectly competitive sellers of signals, the equilibrium is predicted by Spence's model. In contrast, when the market is served by a monopolist, the equilibrium is predicted by the current model, which suggests that when the buyer's signaling incentive is relatively strong, monopoly can yield higher social welfare than a competitive market. Thus, promoting competition in a signaling good market is not necessarily socially beneficial.

6 Discussion

In this section, we address some remaining technical points of the paper. First, we study the equilibrium selection in the unobserved case. Then, we explore some extensions. Specifically, we consider the cases of nonessential signals and unproductive signals and the case in which the seller is partially profit-maximizing.

6.1 Equilibrium selection in the unobserved case

In the unobserved case, we focus on the seller-optimal separating equilibrium, which has the following important properties. Among all separating equilibria of the unobserved case, this equilibrium leads to the highest payoff for the school, the largest market coverage (i.e., the lowest cutoff type), and the lowest education level for every participating type. In addition, the equilibrium is the Riley outcome of the unobserved case, as the cutoff type θ_0^o chooses the "full-information" optimum $z^*(\theta_0^o)$. In particular, if one replaces $\underline{\theta}$ with θ_0^o and $C(z,\theta)$ with $G(z,\theta)$ in Spence's game, then the seller-optimal separating equilibrium is indeed the least-cost separating equilibrium. Finally, under certain conditions, this equilibrium is also the unique continuous equilibrium. A formal argument is provided in Appendix C.

However, under some conditions, there may also exist a continuum of pooling equilibria and separating equilibria. In this regard, we propose a novel refinement, *quasi-divinity*,[12] to refine the set of equilibria. The definition of quasi-divinity is given as follows.

[12] I am indebted to Prof. John Riley for his exceptional doctoral class at UCLA that inspired this concept.

Definition 1. *In the unobserved case, an equilibrium satisfies quasi-divinity if there does not exist an off-path signal* $\hat{z}$*, a receiver's response* $\hat{w}$*, and a positive-measure subset* $\hat{\Theta} \subset [\underline{\theta}, \bar{\theta}]$*, satisfying the following conditions:*

(i) The allocation of signals

$$z^d(\theta) = \begin{cases} \hat{z} & \text{if } \theta \in \hat{\Theta} \\ z^u(\theta) & \text{otherwise} \end{cases}$$

is nondecreasing in θ*.*

(ii) $\hat{w} - G(\hat{z}, \theta) > W^u(z^u(\theta)) - G(z^u(\theta), \theta) = J^u(z^u(\theta), \theta)$ *if and only if* $\theta \in \hat{\Theta}$*.*

(iii) $\hat{w} < \mathbb{E}^{\hat{z}}[Q(\hat{z}, \theta) | \theta \in \hat{\Theta}]$*, with the expectation formed under any quasi-divine belief, i.e., any receiver's posterior belief that has a distribution function* $F^{\hat{z}}$ *with* $\text{supp}(F^{\hat{z}}) = \hat{\Theta}$*.*

In other words, Definition 1 states that an equilibrium fails quasi-divinity if there exists an off-path education level and a wage such that the school can make a profitable deviation by choosing some tuition scheme under which a subset of worker types is willing to choose the off-path education level for that wage, whereas all the other types prefer their equilibrium outcomes, and that the employers are willing to offer that wage for that education level, so long as they believe that they are facing a type from that subset, no matter how pessimistic such a belief is. It is worth noting that a deviating type $\theta \in \hat{\Theta}$ may be worse off than in the original equilibrium, whereas he prefers the outcome $(\hat{z}, \hat{w})$ to any other outcome $(z, W^u(z))$ under the new tuition scheme chosen by the school.

The idea of quasi-divinity is as follows. By choosing the off-path signal, the worker sends an implicit message to the employers: "Although you cannot observe the tuition scheme, it would be strictly profitable for you to offer me $\hat{w}$. This is because under the tuition scheme that the school *actually* offers, the set of types who prefer $(\hat{z}, \hat{w})$ to any other pair $(z, W^u(z))$ is $\hat{\Theta}$, and for any quasi-divine belief you may have, you will receive a payoff strictly higher than your equilibrium payoff, i.e., $\mathbb{E}^{\hat{z}}[Q(\hat{z}, \theta) | \theta \in \hat{\Theta}] - \hat{w} > 0$." Anticipating the hypothetical speech by the worker and that the employers would think it through, the school has indeed an incentive to choose some tuition scheme to implement $z^d(\theta)$ given the new wage, as doing so can yield a strictly higher profit while making the worker's statement truthful. Specifically, the monotonicity of $z^d(\theta)$ ensures that the new allocation is implementable and that $\hat{\Theta}$ has a positive measure, combined with condition (ii) of Definition 1, ensures that the school will be strictly better-off than in the original equilibrium, should the employers offer $\hat{w}$ for $\hat{z}$.

Quasi-divinity is closely related to the standard refinements for signaling games, such as universal divinity (Banks and Sobel, 1987) and the Intuitive Criterion (Cho and Kreps, 1987). The key difference is that quasi-divinity directly examines the receiver's responses off the equilibrium path, instead of restricting the off-path beliefs. The critical test is whether there is an off-path signal and a subset of buyer types such that the receiver has a dominant response in terms of quasi-divine beliefs, and the seller can strictly profit by inducing only those types to choose the off-path signal. Furthermore, quasi-divine belief is less restrictive than divine belief, as the latter includes beliefs that place probability one on a single type. But in our continuous-type setting, any response that yields a strictly higher virtual surplus than the equilibrium level at some type also makes it more profitable at each type in a small neighborhood. In this regard, quasi-divinity is more applicable to the current model.

The next theorem indicates that within the set of the equilibria where the allocation $z^u(\theta)$ is piecewise continuous (or simply *equilibria*), the seller-optimal separating equilibrium is the unique equilibrium (outcome) that satisfies quasi-divinity. To summarize,

Theorem 4. *In the unobserved case, the unique equilibrium that satisfies quasi-divinity is the seller-optimal separating equilibrium.*

We now sketch the proof and provide details in Appendix B. There are two possibilities. First, if the highest type $\bar{\theta}$ belongs to a pooling interval, then by the single-crossing property, for sufficiently high types, there exists a $\hat{z} > z^u(\bar{\theta})$ and a dominant wage $\hat{w}$ with respect to quasi-divine beliefs such that the school can profitably induce those types to deviate to $\hat{z}$. Suppose now $\bar{\theta}$ belongs to a separating interval $[\theta_1, \bar{\theta}]$ for some $\theta_1 \geq \theta_0^u$. If the equilibrium is not the seller-optimal separating equilibrium, then we have that $z^u(\theta)$ is discontinuous at θ_1 with $z^u(\theta_1) > z^*(\theta_1)$. Again, by the single-crossing property, for all types in a neighborhood of θ_1, there exists a $\hat{z}$ slightly lower than $z^u(\theta_1)$ and a dominant wage such that the school can profitably induce those types to deviate to $\hat{z}$. Finally, for the seller-optimal separating equilibrium, since the cutoff type θ_0^o has achieved the full-information optimum $z^*(\theta_0^o)$, for sufficiently pessimistic quasi-divine beliefs, there does not exist any off-path signal to which the school can profitably induce some types to deviate. That is, the seller-optimal separating equilibrium is the unique equilibrium that satisfies quasi-divinity. In particular, for discrete types, since each type has a positive measure, quasi-divinity can be applied by replacing $\hat{\Theta}$ with a single type and the support of $F^{\hat{z}}$ with a singleton. It can be shown that the adapted procedure is essentially the same as classic refinements such as universal divinity, and that the seller-optimal separating equilibrium is uniquely selected, as in the two-type example.

6.2 Nonessential signals

Here, we relax the model assumption that $Q(0, \theta) \equiv 0$ and assume instead that $Q_\theta > 0$ for all $(z, \theta) \in \mathbb{R}_+ \times [\underline{\theta}, \bar{\theta}]$, with $Q(0, \underline{\theta})$ normalized to 0. We start with the observed case.

Suppose $z^*(\underline{\theta}) > 0$, then the allocation given by (4.1) also constitutes the seller-optimal equilibrium of this case. To show this, assume that for all z outside the range of $z^o(\theta)$, the school charges prohibitively high prices and the labor market has the worst belief such that the worker's outside option has utility $Q(0, \underline{\theta}) = 0$. It follows that $z^o(\theta)$ with $\theta_0^o = \underline{\theta}$ solves the school's problem and yields the highest possible equilibrium payoff for the school.

Now consider the case in which $\inf\{\theta | z^*(\theta) > 0\} > \underline{\theta}$. Denote the infimum $\underline{\theta}_0$, and define

$$\bar{\Pi}^o := F(\underline{\theta}_0)\mathbb{E}[Q(0, \theta) | \theta \leq \underline{\theta}_0] + \int_{\underline{\theta}_0}^{\bar{\theta}} J^o(\bar{z}(\theta), \theta) dF(\theta),$$

where $\bar{z}(\theta)$ is given as in Section 4.1. It can be shown that for any fixed $\delta > 0$, there exists a subgame associated with some tuition scheme in which the school's equilibrium payoff is greater than $\bar{\Pi}^o - \delta$ (see Lemma 2 in Section B). This implies that if an equilibrium exists, then there is an equilibrium in which $\Pi^o \geq \bar{\Pi}^o - \delta$.

Thus, to achieve the seller-optimal equilibrium (as $\delta \to 0$), we allow the school to charge a fixed fee for zero education, as if the school could sell the worker a certification with no information disclosure as in Lizzeri (1999). Observing that the worker paid the fixed fee, the labor market receives a simple message that the worker is certified by the school and, thus, offers a wage equal to the average productivity of those who paid the fee. Assume that the labor market regards a worker with neither education nor a certification as the lowest type. Thus, the optimal cutoff type is $\theta_0^o = \underline{\theta}_0$, and the optimal fixed fee equals $\mathbb{E}[Q(0, \theta) | \theta \leq \underline{\theta}_0]$ such that any type $\theta \leq \underline{\theta}_0$ will be indifferent. Then, the allocation given by (4.1), combined with the fixed fee, leads to the seller-optimal equilibrium such that $\Pi^o = \bar{\Pi}^o$. Note that in the equilibrium, the market is fully covered and consists of two segments:

(a) The certification segment, $[\underline{\theta}, \theta_0^o]$, where the worker pays a fixed fee for zero education.

(b) The education segment, $(\theta_0^o, \bar{\theta}]$, where the worker purchases a positive education level.

As in Lizzeri (1999), the certifier reveals nothing about the agent's type and extracts all the information rents. However, whereas this extreme equilibrium outcome is sustained by a particular belief in our model, it is the unique equilibrium outcome under certain conditions in Lizzeri's (Lizzeri, 1999, Theorem 3). This is because in contrast to our model, in Lizzeri's, the certifier can truthfully reveal the agent's type at zero cost and, thus, can always induce higher types to participate by revealing the highest type with relatively high probability and the other types with sufficiently low probability.

Now, we turn to the unobserved case. Given the nature of the information structure, we shut down signaling through certification (money burning); thus, again, $T(0) = 0$. Suppose an equilibrium exists, then the school's expected profit is given by

$$\Pi^u = \int_{\theta_0^u}^{\bar{\theta}} J^u(z^u(\theta), \theta) dF(\theta) - [1 - F(\theta_0^u)]\mathbb{E}[Q(0, \theta)|\theta \leq \theta_0^u],$$

where the worker's reservation utility, $\mathbb{E}[Q(0, \theta)|\theta \leq \theta_0^u]$, is constant given the market belief. We claim that if $\theta_0^u > \underline{\theta}$, then $z^u(\theta)$ jumps discontinuously at θ_0^u with $z^u(\theta_0^u) > z^*(\theta_0^u)$ (see Lemma 3 in Section B). Then, analogous to Theorems 2 and 4, we have the following result.

Theorem $2'$**.** *In the unobserved case, there exists a unique equilibrium that satisfies quasi-divinity in which* $z^u(\theta)$ *is continuous and increasing on* $[\underline{\theta}, \bar{\theta}]$ *and satisfies (4.3) on* $(\underline{\theta}, \bar{\theta}]$*, with the initial condition* $(\theta_0^u, z^u(\theta_0^u)) = (\underline{\theta}, z^*(\underline{\theta}))$*.*

Similarly, Theorem $2'$ indicates that the unique equilibrium that satisfies quasi-divinity is the Riley outcome, that is, the separating equilibrium in which the cutoff type θ_0^u chooses the full-information optimum $z^*(\theta_0^u)$. Under nonessential education, if $\theta_0^u > \underline{\theta}$, then θ_0^u must choose a higher education level than $z^*(\theta_0^u)$ such that the school has no incentive to induce lower types to mimic θ_0^u. This in turn implies that in the Riley outcome, $\theta_0^u = \underline{\theta}$, and thus, $z^u(\underline{\theta}) = z^*(\underline{\theta}) \geq z^o(\underline{\theta})$. Then, analogous to Theorem 3, we have the following result.

Theorem $3'$**.** *In contrast with the observed case, the worker acquires more education in the unobserved case. Specifically,* $z^u(\theta) \geq z^o(\theta)$ *on* $[\underline{\theta}, \bar{\theta}]$*, with strict inequality for* $\theta > \underline{\theta}$*.*

Then, one can show analogously that in the unobserved case, the school receives a lower expected profit while the worker obtains higher utility, and the tuition is lower than in the observed case (as stated by Corollaries 1 and 2, and Proposition 2, respectively).

6.3 Unproductive signals

Now, we consider the case where education is unproductive, i.e., $Q_z(z, \theta) \equiv 0$, which can be regarded as a limit case of nonessential education. Accordingly, we rewrite the productivity function as $Q(\theta)$ and assume that $Q_\theta > 0$ for all (z, θ) with $Q(\underline{\theta})$ normalized to 0.

In the observed case, because $z^*(\theta) \equiv 0$, the seller-optimal equilibrium contains only the certification segment: $z^o(\theta) \equiv 0$, and the school charges a certification fee equal to $\mathbb{E}[Q(\theta)]$. That is, the school fully extracts the surplus with no information disclosure at all.

In the unobserved case, we have that both Theorems 2′ and 3′ still hold here; thus, the implications for tuition, and the school and worker's payoffs remain unchanged. Moreover, because $z^u(\theta) > z^o(\theta) = z^{fb}(\theta)$ on $(\underline{\theta}, \bar{\theta}]$, the unobserved case unambiguously yields a lower social welfare than the observed case. In addition, it is easy to show that $z^u(\theta) \leq z^s(\theta)$ with strict inequality on $(\underline{\theta}, \bar{\theta}]$. Therefore, Spence's game yields lower social welfare than both the observed and unobserved cases.

6.4 Partially profit-maximizing seller

In the application of job market signaling, we assume that the school maximizes its expected profit. In reality, however, schools are typically not pure profit-maximizers. To this end, we study the school's pricing strategy when its objective is a weighted average of its profit and the worker's utility. Formally, given a wage schedule W, the school solves

$$\max_{z(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[W(z) - C(z,\theta) - U(\theta)\right] dF(\theta) + \mu \int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta),$$

where $\mu \in (0, 1]$ denotes the relative Pareto weight. In particular, $\mu = 0$ leads to the original model in which the school maximizes its profit; $\mu = 1$ means that the school maximizes the joint surplus of the two parties. This variant thus only differs in that $G(z, \theta)$ is replaced by

$$G(z,\theta;\mu) \coloneqq C(z,\theta) + (1-\mu)\frac{1 - F(\theta)}{f(\theta)} \left[-C_\theta(z,\theta)\right].$$

It is readily confirmed that for all $\mu \in (0, 1]$, $G_z > 0$, $G_{zz} > k$ for some $k > 0$, and $G_{z\theta} < 0$. Thus, all the results in Section 4 remain unchanged up to μ. It is easy to show that in both the observed and unobserved cases, as μ increases, the worker chooses more education and obtains higher utility. Intuitively, as the school places greater weight on the worker's payoff, the worker can receive more education, thereby extracting more rents.

However, an increase in μ has qualitatively different welfare implications in the observed and unobserved cases. In the observed case, as μ increases, the screening distortion decreases and social welfare increases. In particular, when the school maximizes the joint surplus of the worker and itself ($\mu = 1$), the outcome is socially optimal. In the unobserved case, however, as μ increases from 0 to 1, the equilibrium switches from that of the original unobserved case to that of Spence's game. As a result, the welfare implication is ambiguous. In particular, if the original unobserved case yields higher social welfare than Spence's game, then a profit-maximizing school might be more socially beneficial than a school that maximizes the joint surplus of itself and its students since in the latter case, the school charges such low tuition that there is significant over-education.

7 Conclusion

In this paper, we developed classic signaling models by letting a strategic player affect the cost of signaling. A seller chooses a price scheme for a good, and a buyer with a hidden type chooses how much to purchase as a signal to receivers. The equilibrium depends critically on whether receivers observe the price scheme. In the observed case, the seller internalizes signaling in screening, causing a downward distortion. However, such distortion is smaller than that in the case where receivers also observe the buyer's type. In the unobserved case, the buyer is more sensitive to price changes than in the observed case. This leads to a more elastic demand for signals and provides the seller with an incentive to cut prices. To refine the set of equilibria, we proposed a new refinement, quasi-divinity. In equilibrium, the buyer chooses a higher quantity and obtains higher utility than in the observed case, whereas the seller gains lower profits than in the observed case. We also showed that price transparency can be socially beneficial and social welfare can be higher in a monopoly market than in a competitive market when the buyer's signaling incentive is relatively strong. Our framework can be applied to schools choosing tuition, retailers selling luxury goods, media companies selling advertising messages, and to other vertical markets in which signaling prevails.

A Appendix

A Proofs for Sections 4 and 5

Proof of Theorem 1

Proof. Given our assumptions, $J^o(z,\theta)$ is strictly concave in z. It follows from Toikka (2011, Theorem 4.4) that the school's problem boils down to pointwise maximization for $\bar{J}^o(z,\theta)$. In addition, $\bar{J}^o(z,\theta)$ is also strictly concave in z and thus has a unique maximizer on $\mathbb{R}+$, denoted $\bar{z}(\theta)$, which is continuous and nondecreasing on $[\underline{\theta},\bar{\theta}]$. Moreover, if $\bar{z}(\theta) \neq z^*(\theta)$ for some θ, then $H(\bar{z}(\theta),\theta)$ and $I(\bar{z}(\theta),\theta)$ differ in a neighborhood of θ, in which $I_\theta(\bar{z}(\theta),\theta)$ is flat; thus, $\bar{z}(\theta)$ is constant in this neighborhood. That is, if $\bar{z}(\theta) \neq z^*(\theta)$, then θ belongs to a pooling interval; outside these intervals, $\bar{z}(\theta) = z^*(\theta)$. Note that $J^o_\theta(z,\theta) > 0$ if $z > 0$, and that $J^o(0,\theta) \equiv 0$. It follows from the envelope theorem that $J^o(\bar{z}(\theta),\theta)$ is nondecreasing in θ and nonnegative. Thus, the cutoff type θ^o_0 is either the maximal root of $J^o(\bar{z}(\theta),\theta) = 0$ if it exists or $\underline{\theta}$ otherwise. In particular, if $\theta^o_0 > \underline{\theta}$, then we have $\bar{z}(\theta^o_0) = z^*(\theta^o_0) = 0$. Suppose not, then $\bar{z}(\theta^o_0) > z^*(\theta^o_0) > 0$, as $J^o(z,\theta)$ is strictly concave in z; thus, $J^o(z,\theta^o_0) > 0$ on $(0,\bar{z}(\theta^o_0))$. Then, the school can profit by assigning any $z \in (0,\bar{z}(\theta^o_0))$ to some neighborhood of θ^o_0 with the monotonicity still holding, a contradiction. It follows that θ^o_0 is either the maximal root of $\bar{z}(\theta) = 0$ if it exists or $\underline{\theta}$ otherwise. Moreover, if $z^*(\underline{\theta}) > 0$, then $J^o(z^*(\underline{\theta}),\underline{\theta}) > 0$ because $J^o(0,\theta) \equiv 0$ and $J^o(z,\theta)$ is strictly concave in z. Since $J^o(z^*(\theta),\theta)$ is increasing in θ, we have $J^o(z^*(\theta),\theta) > 0$ for all θ; thus, $z^*(\theta) > 0$ on $[\underline{\theta},\bar{\theta}]$. This implies that if $z^*(\underline{\theta}) > 0$, then we have $\theta^o_0 = \underline{\theta}$ and $\bar{z}(\underline{\theta}) \leq z^*(\underline{\theta})$. This completes the characterizations of $z^o(\theta)$ and θ^o_0. Then, the characterizations of T^o and W^o follow immediately. Thus, the theorem is proven. □

Proof of Proposition 1

Proof. Note that $J^o_z(z,\theta)$ is lower than $S_z(z,\theta)$, holding weakly on the boundary. Thus, we have $z^*(\theta) \leq z^{fb}(\theta)$ on $(\underline{\theta},\bar{\theta}]$, with equality holding at $\bar{\theta}$ only. In addition, $J^o_z(z,\cdot)$ must be increasing for θ close to $\bar{\theta}$ since $J^o_{z\theta}(z,\bar{\theta}) > 0$. Thus, $H(z,\theta)$ coincides with $I(z,\theta)$ in the left neighborhood of $\bar{\theta}$, meaning that $z^o(\theta)$ is increasing near $\bar{\theta}$, and thus, $z^o(\bar{\theta}) = z^{fb}(\bar{\theta})$. Note further that any θ satisfying $z^*(\theta) < z^o(\theta)$ belongs to some pooling interval $[\alpha,\beta]$ such that $z^o(\theta) = \min\{z^*(\alpha), z^*(\beta)\}$.[13] Because $z^{fb}(\theta) \geq z^*(\theta)$, $z^o(\theta) \leq z^{fb}(\theta)$ on $[\alpha,\beta]$. Outside these pooling intervals, $z^o(\theta) = z^*(\theta) \leq z^{fb}(\theta)$. Note that $z^*(\theta) < z^{fb}(\theta)$ on $(\underline{\theta},\bar{\theta})$. It follows that $z^o(\theta) \leq z^{fb}(\theta)$ on $(\underline{\theta},\bar{\theta}]$, with equality holding at $\bar{\theta}$ only. Thus, the proposition is proven. □

[13] It is possible that $\underline{\theta}$ belongs to a pooling interval with $z^o(\underline{\theta}) < z^*(\underline{\theta})$.

Proof of Theorem 2

Proof. Let $J(\theta, \hat{\theta}, z) := Q(z, \hat{\theta}) - G(z, \theta)$ be the virtual surplus of type θ if he is believed as type $\hat{\theta}$ and chooses education level z. In particular, $J(\theta, \theta, z) \equiv J^o(z, \theta)$. Given the model assumptions, we have $J_{zz}(\theta, \hat{\theta}, z) < -k$ for some $k > 0$, $J_{\hat{\theta}}(\theta, \hat{\theta}, z) = Q_\theta(z, \hat{\theta}) > 0$ if $z > 0$, $J_{z\theta}(\theta, \hat{\theta}, z) = -G_{z\theta}(z, \theta) > 0$, and $J_z(\theta, \hat{\theta}, z)/J_{\hat{\theta}}(\theta, \hat{\theta}, z)$ is increasing in θ. For the moment, we restrict our attention to the domain $[\theta_0^o, \bar{\theta}] \times [\theta_0^o, \bar{\theta}] \cup \{\underline{\theta}\} \times \mathbb{R}_+$, which incurs no loss of generality if $\theta_0^o = \underline{\theta}$. By Mailath and von Thadden (2013, Theorems 2, 5 and 6), there exists a separating equilibrium in which $z^u(\theta)$ is continuous and increasing on $[\theta_0^o, \bar{\theta}]$, and satisfies (4.3) on $(\theta_0^o, \bar{\theta}]$ with $z^u(\theta_0^o) = z^*(\theta_0^o)$; $z^u(\theta)$ is unique given (4.3) and the initial condition. It follows from (4.3) that the inverse of $z^u(\theta)$, $\theta^u(z)$, is also differentiable on $(\theta_0^o, \bar{\theta}]$. In terms of any off-path $z > 0$, assume simply that the labor market holds the worst belief $\underline{\theta}$. Then, the wage schedule and the tuition scheme can be characterized immediately. Now, we consider the full domain $[\underline{\theta}, \bar{\theta}]^2 \times \mathbb{R}_+$ if $\theta_0^o > \underline{\theta}$. Clearly, the incentive of any type $\theta \geq \theta_0^o$ will not be affected. It thus remains to examine whether the school can profit by assigning some $z > 0$ to some type $\theta \in [\underline{\theta}, \theta_0^o)$. Fix such θ. For any off-path $z > 0$, $J(\theta, \underline{\theta}, z) \leq J(\theta_0^o, \underline{\theta}, z) \leq 0$, thus a profitable deviation does not exist. On the other hand, if $z = z^u(\theta')$ for some $\theta' \geq \theta_0^o$, then by $G_{z\theta} < 0$, $J(\theta, \theta', z) - J(\theta, \theta, 0) \leq J(\theta_0^o, \theta', z) - J(\theta_0^o, \theta_0^o, 0) \leq 0$. The second inequality is due to that $z^*(\theta_0^o) = 0$ if $\theta_0^o > \underline{\theta}$ and type θ_0^o's IC constraint. Thus, it is indeed optimal for the school to exclude all $\theta \in [\underline{\theta}, \theta_0^o)$. This proves the existence of such an equilibrium.

Finally, we show that the above equilibrium is the most profitable for the school among all separating equilibria. By Corollary 3 in Appendix C, a separating equilibrium's outcome is uniquely pinned down by the cutoff type θ_0^u. Moreover, in each separating equilibrium of the unobserved case, $\theta_0^u \geq \theta_0^o$. Suppose in addition to θ_0^o, there exists an equilibrium cutoff type $\hat{\theta}_0^u > \theta_0^o$. Let $z_1^u(\theta)$ and $z_2^u(\theta)$ be the equilibrium education functions associated with θ_0^o and $\hat{\theta}_0^u$, respectively. By Mailath and von Thadden (2013, Theorems 2), we have that both $z_1^u(\theta)$ and $z_2^u(\theta)$ satisfy (4.3) on $(\theta_0^u, \bar{\theta}]$. Moreover, by Corollary 3, $z^u(\hat{\theta}_0^u) > z^*(\theta_0^o)$. It follows from Hartman (1964, Corollary 4.2, page 27) that $z_2^u(\theta) > z_1^u(\theta)$ on $[\hat{\theta}_0^u, \bar{\theta}]$. By the proof of Theorem 3, $z_2^u(\theta) > z_1^u(\theta) \geq z^*(\theta)$ on $[\hat{\theta}_0^u, \bar{\theta}]$. Note that $J^u(z^u(\theta), \theta) = J^o(z^u(\theta), \theta)$ on $[\hat{\theta}_0^u, \bar{\theta}]$. Since $J^o(z, \theta)$ is strictly concave in z, $J^u(z_1^u(\theta), \theta) > J^u(z_2^u(\theta), \theta)$ on $(\theta_0^o, \bar{\theta}]$. Let Π_1^u and Π_2^u denote the school's equilibrium payoff associated with θ_0^o and $\hat{\theta}_0^u$, respectively. Thus,

$$\Pi_1^u - \Pi_2^u = \int_{\theta_0^o}^{\bar{\theta}} J^u(z_1^u(\theta), \theta) dF(\theta) - \int_{\hat{\theta}_0^u}^{\bar{\theta}} J^u(z_2^u(\theta), \theta) dF(\theta) > 0.$$

That is, the separating equilibrium with the cutoff type θ_0^o yields the highest payoff for the school among all separating equilibria. Thus, the theorem is proven. □

Proof of Theorem 3

Proof. We first show that in each separating equilibrium, $z^u(\theta) \geq z^*(\theta)$ on $[\theta_0^u, \bar{\theta}]$ with strict inequality for $\theta > \theta_0^u$. Since $J^u(z^u(\theta), \theta) = J^o(z^u(\theta), \theta)$ and $J^o(z, \theta)$ is strictly concave in z, it suffices to show that $J_z^o(z^u(\theta), \theta) \leq 0$ with strict inequality for $\theta > \theta_0^u$. From Corollary 3, $z^u(\theta)$ satisfies (4.3) on $(\theta_0^u, \bar{\theta}]$. Then, for each $\theta \geq \theta_0^u$, we have

$$\begin{aligned} J_z^o(z^u(\theta), \theta) &= Q_z(z^u(\theta), \theta) - G_z(z^u(\theta), \theta) \\ &\leq Q_z(z^u(\theta), \theta) + Q_\theta(z^u(\theta), \theta)\theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta), \theta) = 0. \end{aligned}$$

The inequality is because $z^u(\theta)$ is nondecreasing; the last equality is due to (4.3). Moreover, for $\theta > \theta_0^u$, the second term in (4.3) is positive; thus, the above inequality becomes strict.

Then, we prove that in each separating equilibrium, $z^u(\theta) \geq z^o(\theta)$ on $[\theta_0^u, \bar{\theta}]$ with strict inequality for $\theta > \theta_0^u$. By Corollary 3, we have $z^u(\theta_0^u) \geq \max\{z^*(\theta_0^u), z^o(\theta_0^u)\}$. Suppose there exists a $\tilde{\theta} \in (\theta_0^u, \bar{\theta}]$ such that $z^u(\tilde{\theta}) \leq z^o(\tilde{\theta})$, then $\tilde{\theta}$ belongs to a pooling interval with respect to $z^o(\theta)$ since $z^u(\theta) > z^*(\theta)$ on $(\theta_0^u, \bar{\theta}]$. Let α be the left end of this interval. Since $z^u(\theta)$ is increasing on $[\theta_0^u, \bar{\theta}]$, $z^u(\theta) < z^o(\theta)$ on $[\alpha, \tilde{\theta})$. This implies that $\alpha > \theta_0^u$ since $z^u(\theta_0^u) \geq z^o(\theta_0^u)$. Then, by the continuity of $z^o(\theta)$, we have $z^*(\alpha) = z^o(\alpha) > z^u(\alpha)$, a contradiction. Therefore, $z^u(\theta) \geq z^o(\theta)$ on $[\theta_0^u, \bar{\theta}]$ with strict inequality for $\theta > \theta_0^u$. In particular, in the seller-optimal separating equilibrium, we have $\theta_0^u = \theta_0^o$. It follows that $z^u(\theta) \geq z^o(\theta)$ on $[\underline{\theta}, \bar{\theta}]$ with strict inequality for $\theta > \theta_0^o$. Thus, the theorem is proven. □

Proof of Proposition 2

Proof. For each $z \in [z^*(\theta_0^o), z^*(\bar{\theta})]$, define $\theta^o(z) := \inf\{\theta | z^o(\theta) \geq z\}$. Since $C_\theta = 0$ if $z = 0$, by (4.2) and (4.4), for each $z \in [z^*(\theta_0^o), z^*(\bar{\theta})]$, $T^o(z) - T^u(z)$ equals

$$W^o(z) - W^u(z) - [C(z, \theta^o(z)) - C(z, \theta^u(z))] + \int_{\underline{\theta}}^{\theta^o(z)} C_\theta(z^o(s), s)ds - \int_{\underline{\theta}}^{\theta^u(z)} C_\theta(z^u(s), s)ds$$

$$\geq W^o(z) - W^u(z) - [C(z, \theta^o(z)) - C(z, \theta^u(z))] + \int_{\theta^u(z)}^{\theta^o(z)} C_\theta(z^o(s), s)ds$$

$$= W^o(z) - W^u(z) + \int_{\theta^u(z)}^{\theta^o(z)} [C_\theta(z^o(s), s) - C_\theta(z, s)]ds$$

$$\geq W^o(z) - W^u(z).$$

The first inequality dues to Theorem 3 and that $C_\theta < 0$ if $z > 0$; the second inequality dues to that $C_{z\theta} < 0$, $\theta^o(z) \geq \theta^u(z)$ and $z^o(\theta) \leq z$ for all $\theta \in [\theta^u(z), \theta^o(z)]$. Moreover, since wage equals the worker's expected productivity, we have $W^o(z) \geq W^u(z)$ on $[z^*(\theta_0^o), z^*(\bar{\theta})]$ with strict inequality for $z > z^*(\theta_0^o)$. Thus, the proposition is proven. □

Proof of Proposition 3

Proof. We only need to prove that $z^u(\theta) < z^s(\theta)$ on $(\theta_0^o, \bar{\theta}]$. Rearranging (2.4) and (4.3), we can derive $z^s(\theta)$ and $z^u(\theta)$ through the initial value problems:

$$z^{s\prime} = \frac{Q_\theta(z,\theta)}{C_z(z,\theta) - Q_z(z,\theta)} \quad \text{and} \quad z^{u\prime} = \frac{Q_\theta(z,\theta)}{G_z(z,\theta) - Q_z(z,\theta)}$$

with the respective initial points $(\theta_0^o, z^s(\theta_0^o))$ and $(\theta_0^o, z^u(\theta_0^o))$. Note that $C_z(z,\theta) \leq G_z(z,\theta)$ with strict inequality for $\theta < \bar{\theta}$. Further, by Proposition 1 and Theorem 2, $z^s(\theta_0^o) \geq z^u(\theta_0^o)$. It then follows from Hartman (1964, Corollary 4.2, page 27) that $z^s(\theta) \geq z^u(\theta)$ on $[\theta_0^o, \bar{\theta}]$ with strict inequality for $\theta > \theta_0^o$. Thus, the proposition is proven. □

Proof of a generalized version of Proposition 4

Here, we consider a generalized version of Mussa and Rosen's game in Section 5. Suppose after the worker chooses education, the labor market can observe the worker's ability with probability $p \in (0, 1]$. For example, the worker takes a test in school. With probability p, the test is perfectly informative and thus reveals the worker's ability; otherwise, the test is completely uninformative about the worker's ability. Thus, p measures the informativeness of the test; in particular, $p = 1$ corresponds to Mussa and Rosen's game. For simplicity, we assume that p is independent of the worker's ability and education level. From the worker's (and the school's) perspective, his expected wage for education level z is given by

$$\mathbb{E}[W(z)] = pQ(z,\theta) + (1-p)\mathbb{E}_\theta[Q(z,\theta)],$$

where the second expectation stands for the labor market's expectation of the worker's productivity. Then, given some tuition scheme T, the worker's expected utility is given by $u(z,\theta) = \mathbb{E}[W(z)] - C(z,\theta) - T(z)$. Analogous to Section 5, the virtual surplus is now

$$J^{mr}(z,\theta) := S(z,\theta) - \frac{1-F(\theta)}{f(\theta)} \left[pQ_\theta(z,\theta) - C_\theta(z,\theta)\right].$$

Assume that both $J^o(z,\theta)$ and $J^{mr}(z,\theta)$ are regular. Therefore, $z^{mr}(\theta)$ and θ_0^{mr} can be solved by pointwise maximization for $J^{mr}(z,\theta)$. On the extensive margin, if $\theta_0^o > \underline{\theta}$, then $\theta_0^{mr} > \theta_0^o$. On the intensive margin, if $Q_{z\theta} > 0$ on $[0, z^{fb}(\bar{\theta})]$, then $z^{mr}(\theta) \leq z^o(\theta)$ with strict inequality on $[\theta_0^o, \bar{\theta})$. Clearly, these differences will expand as p increases.

Indeed, the optimal allocation $z^{mr}(\theta)$ varies continuously from that of the observed case to that of Mussa and Rosen's game as p increases from 0 to 1 (i.e., as the test becomes more informative). In other words, the screening distortion is intensified if signaling is attenuated. In summary, we have the following generalized version of Proposition 4.

Proposition 4′**.** *Suppose both* $J^o(z,\theta)$ *and* $J^{mr}(z,\theta)$ *are regular and* $Q_{z\theta} > 0$ *on* $[0, z^{fb}(\bar{\theta})]$*, then under-education will be greater if signaling is attenuated. Specifically,* $z^{mr}(\theta) \leq z^o(\theta)$*, with strict inequality on* $[\theta_0^o, \bar{\theta})$*. If* $\theta_0^o > \underline{\theta}$*, then* $\theta_0^{mr} > \theta_0^o > \underline{\theta}$*. Moreover, the school's expected profit and welfare are strictly higher in the observed case than in the generalized Mussa and Rosen game. These differences will expand as* p *increases (i.e., as signaling is attenuated).*

B Proofs for Section 6

Proof of Theorem 4

Proof. According to Appendix C, there are two cases to consider. First, suppose $\bar{\theta}$ belongs to a pooling interval $[\theta_1, \bar{\theta}]$, then fix a type α close to $\bar{\theta}$, and choose some $\hat{z} > \tilde{z}$ such that

$$Q(\hat{z},\alpha) - G(\hat{z},\alpha) = W^u(\tilde{z}) - G(\tilde{z},\alpha),$$

where $\tilde{z} > 0$ is the equilibrium education level for $\theta \geq \theta_1$ and $W^u(\tilde{z}) = \mathbb{E}[Q(\tilde{z},\theta)|\theta \geq \theta_1]$. It is clear that such $\hat{z}$ exists. Let $\hat{w} = Q(\hat{z},\alpha)$. For any $\theta > \alpha$, we have

$$\hat{w} - G(\hat{z},\theta) - [W^u(\tilde{z}) - G(\tilde{z},\theta)] > \hat{w} - W^u(\tilde{z}) - [G(\hat{z},\alpha) - G(\tilde{z},\alpha)] = 0.$$

The inequality is due to that $G_{z\theta} < 0$ and $\hat{z} > \tilde{z}$. Analogously, for any $\theta \leq \alpha$, we have

$$\hat{w} - G(\hat{z},\theta) - [W^u(z^u(\theta)) - G(z^u(\theta),\theta)] \leq \hat{w} - G(\hat{z},\theta) - [W^u(\tilde{z}) - G(\tilde{z},\theta)] \leq 0.$$

The first inequality is due to the optimality of $z^u(\theta)$. Then, define $\hat{\Theta} := (\alpha, \bar{\theta}]$, and thus,

$$\hat{w} - G(\hat{z},\theta) > W^u(z^u(\theta)) - G(z^u(\theta),\theta) = J^u(z^u(\theta),\theta)$$

if and only if $\theta \in \hat{\Theta}$. Moreover, note that for any quasi-divine belief,

$$\mathbb{E}^{\hat{z}}[Q(\hat{z},\theta)|\theta > \alpha] > Q(\hat{z},\alpha) = \hat{w}.$$

Choose the allocation $z^d(\theta)$ such that $z^d(\theta) = \hat{z}$ if $\theta \in \hat{\Theta}$; $z^d(\theta) = z^u(\theta)$ otherwise. Because $\hat{z} > \tilde{z}$, $z^d(\theta)$ is nondecreasing. It follows from Definition 1 that any equilibrium such that $\bar{\theta}$ belongs to a pooling interval fails quasi-divinity.

Second, suppose $\bar{\theta}$ belongs to a separating interval $[\theta_1, \bar{\theta}]$. There are two cases. First, the equilibrium is not the seller-optimal separating equilibrium. Define $J(\theta, \hat{\theta}, z)$ as in the proof of Theorem 2. If $\theta_1 > \theta_0^u$, then by Lemma 5 in Appendix C, $z^u(\theta)$ is discontinuous at θ_1 and constant in the left neighborhood of θ_1. Fix a type $\alpha < \theta_1$ that is close to θ_1, then

$$J(\alpha,\alpha,z^u(\theta_1)) < J(\alpha,\theta_1,z^u(\theta_1)) \leq J^u(z^u(\alpha),\alpha) < J(\alpha,\alpha,z^u(\alpha)).$$

The first inequality is because $J_{\hat{\theta}} > 0$ if $z > 0$; the second is due to the optimality of $z^u(\alpha)$; the last is because α belongs to a pooling interval with $z^u(\alpha) > 0$ and α is close to θ_1. Then by the intermediate value theorem, there exists a $\hat{z} \in (z^u(\alpha), z^u(\theta_1))$ such that

$$Q(\hat{z}, \alpha) - G(\hat{z}, \alpha) = J^u(z^u(\alpha), \alpha).$$

If $\theta_1 = \theta_0^u$, then $z^u(\alpha) = 0$ and $z^u(\theta_1)$ is the maximal root of $J^o(z, \theta_1) = 0$ by Corollary 3. Let $\hat{z}$ be the maximal root of $J^o(z, \alpha) = 0$. Clearly, $\hat{z} \in (0, z^u(\theta_1))$, and thus it satisfies the above equation. Then, let $\hat{w} = Q(\hat{z}, \alpha)$. Since $G_{z\theta} < 0$, by the envelope theorem,

$$\frac{d}{d\theta}\left[\hat{w} - G(\hat{z}, \theta) - J^u(z^u(\theta), \theta)\right] = G_\theta(z^u(\theta), \theta) - G_\theta(\hat{z}, \theta) > 0 \tag{A.1}$$

if and only if $\theta < \theta_1$. Since α is close to θ_1, there exists a type $\beta \in (\theta_1, \bar{\theta})$ such that

$$\hat{w} - G(\hat{z}, \theta) - J^u(z^u(\theta), \theta) > 0$$

if and only if $\theta \in (\alpha, \beta)$. Then, define $\hat{\Theta} := (\alpha, \beta)$. Note that for any quasi-divine belief,

$$\mathbb{E}^{\hat{z}}[Q(\hat{z}, \theta) | \theta \in \hat{\Theta}] > Q(\hat{z}, \alpha) = \hat{w}.$$

Choose the allocation $z^d(\theta)$ such that $z^d(\theta) = \hat{z}$ if $\theta \in \hat{\Theta}$; $z^d(\theta) = z^u(\theta)$ otherwise. Clearly, $z^d(\theta)$ is nondecreasing. Then, by Definition 1, such an equilibrium fails quasi-divinity too.

It remains to show that the seller-optimal separating equilibrium satisfies quasi-divinity. Suppose not, then there exists an off-path education level $\hat{z}$, a wage $\hat{w}$ and a positive-measure subset $\hat{\Theta}$, satisfying conditions (i) to (iii) of Definition 1. There are two possibilities. First, if $\hat{z} > z^u(\bar{\theta})$, then by the proof of Theorem 3, $\hat{z} > z^*(\theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Note that $\hat{w} < Q(\hat{z}, \bar{\theta})$, as required by condition (iii). Thus, for any θ, we have

$$\hat{w} - G(\hat{z}, \theta) < Q(\hat{z}, \bar{\theta}) - G(\hat{z}, \theta) < Q(z^u(\bar{\theta}), \bar{\theta}) - G(z^u(\bar{\theta}), \theta) \leq J^u(z^u(\theta), \theta).$$

The second inequality is due to that $Q(z, \bar{\theta}) - G(z, \theta)$ is strictly concave in z and thus has a unique maximizer that is smaller than $z^*(\bar{\theta})$, and that $\hat{z} > z^u(\bar{\theta}) > z^*(\bar{\theta})$. The last inequality is due to the optimality of $z^u(\theta)$. Hence, $\hat{\Theta}$ is empty, a contradiction. Second, if $\hat{z} < z^u(\theta_0^u)$, then by Corollary 3, $\theta_0^u = \underline{\theta}$ and $z^u(\underline{\theta}) = z^*(\underline{\theta})$. Since $\hat{\Theta}$ has a positive measure, there exists a type $\alpha > \underline{\theta}$ such that $\hat{w} - G(\hat{z}, \alpha) > J^u(z^u(\alpha), \alpha)$. Moreover, since $\hat{z} < z^u(\underline{\theta})$, the derivative in (A.1) is negative for all $\theta \in [\underline{\theta}, \bar{\theta}]$. This means that $\underline{\theta} \in \hat{\Theta}$, i.e.,

$$\hat{w} - G(\hat{z}, \underline{\theta}) > J^u(z^u(\underline{\theta}), \underline{\theta}) = J^o(z^*(\underline{\theta}), \underline{\theta}) > J^o(\hat{z}, \underline{\theta}).$$

But for sufficiently pessimistic quasi-divine beliefs, we have $|\hat{w} - Q(\hat{z}, \underline{\theta})| < \varepsilon$ for any $\varepsilon > 0$. That is, the LHS of the inequality is bounded above by $J^o(\hat{z}, \underline{\theta}) + \varepsilon$, a contradiction. Thus, this equilibrium satisfies quasi-divinity. In summary, the theorem is proven. □

Lemma 2. *In the observed case, for any $\delta > 0$, there is a subgame associated with some T in which a signaling equilibrium exists with the school's payoff being greater than $\bar{\Pi}^o - \delta$.*

Proof. Since $\underline{\theta}_0 > \underline{\theta}$, $\bar{z}(\theta)$ is increasing in the right neighborhood of $\underline{\theta}_0$. Moreover, note that $J^o(z,\theta) \to Q(0,\theta)$ as $z \to 0$. Thus, given δ, there exists a small $\varepsilon > 0$ such that

$$\int_{\underline{\theta}}^{\theta^o(\varepsilon)} J^o(\varepsilon,\theta)dF(\theta) + \int_{\theta^o(\varepsilon)}^{\bar{\theta}} J^o(\bar{z}(\theta),\theta)dF(\theta) \geq \bar{\Pi}^o - \delta, \tag{A.2}$$

where $\theta^o(\varepsilon)$ is the preimage of $\bar{z}(\theta)$ at ε. Clearly, $\theta^o(\varepsilon) > \underline{\theta}_0$. Then, let the allocation be

$$z^o(\theta) = \begin{cases} \bar{z}(\theta) & \text{if } \theta \geq \theta^o(\varepsilon) \\ \varepsilon & \text{otherwise,} \end{cases}$$

and assume again that for all z outside the range of $z^o(\theta)$, the school charges prohibitively high prices and the labor market holds the worst belief. It is easy to verify that $z^o(\theta)$ and the wage schedule resulted from the above market belief constitute a signaling equilibrium in the corresponding subgame such that the school's payoff is equal to the LHS of (A.2). Thus, the lemma is proven. □

Lemma 3. *In the unobserved case, if $\theta_0^u > \underline{\theta}$, then $z^u(\theta_0^u) > z^*(\theta_0^u) > \lim_{\theta\uparrow\theta_0^u} z^u(\theta) = 0$.*

Proof. Given Π^u, the virtual surplus of θ_0^u should satisfy that

$$-f(\theta_0^u)\left(J^u(z^u(\theta_0^u),\theta_0^u) - \mathbb{E}[Q(0,\theta)|\theta \leq \theta_0^u]\right) \leq 0.$$

In particular, if $\theta_0^u > \underline{\theta}$, then we must have

$$J^u(z^u(\theta_0^u),\theta_0^u) - \mathbb{E}[Q(0,\theta)|\theta \leq \theta_0^u] = 0. \tag{A.3}$$

It follows that $z^u(\theta)$ is discontinuous at θ_0^u. Suppose not, then $W^u(z)$ has an upward jump at 0, since $z^u(\theta)$ is increasing in a small right neighborhood of θ_0^u and $z^u(\theta) \equiv 0$ for $\theta \leq \theta_0^u$. But then the school can profit by assigning a sufficiently small education level $z > 0$ to the types in some neighborhood of θ_0^u, thereby receiving a discrete profit gain, a contradiction. Thus, in each equilibrium with $\theta_0^u > \underline{\theta}$, $z^u(\theta_0^u) > \lim_{\theta\uparrow\theta_0^u} z^u(\theta) = 0$. Since $z^u(\theta)$ is increasing at θ_0^u, $J^u(z^u(\theta_0^u),\theta_0^u) = J^o(z^u(\theta_0^u),\theta_0^u)$. Note that $J^o(0,\theta_0^u) = Q(0,\theta_0^u) > \mathbb{E}[Q(0,\theta)|\theta \leq \theta_0^u]$, as $\theta_0^u > \underline{\theta}$. Since $J^o(z,\theta)$ is strictly concave in z, it follows from (A.3) that $z^u(\theta_0^u) > z^*(\theta_0^u) > 0$. Thus, the lemma is proven. □

C Further discussion on the equilibrium of the unobserved case

Here, we provide a more comprehensive analysis of the equilibrium of the unobserved case. We first establish the continuity of the virtual surplus in any equilibrium.

Lemma 4. *In each equilibrium of the unobserved case, $J^u(z^u(\theta), \theta)$ is continuous in θ.*

Proof. Define $J(\theta, \hat{\theta}, z)$ as in the proof of Theorem 2. Note that $J(\theta, \hat{\theta}, z)$ is continuous and increasing in $\hat{\theta}$ on $[\underline{\theta}, \bar{\theta}]$. It follows from the intermediate value theorem that for each θ, there exists a $\hat{\theta}(\theta) \in [\underline{\theta}, \bar{\theta}]$ such that $J(\theta, \hat{\theta}(\theta), z^u(\theta)) = J^u(z^u(\theta), \theta)$. In particular, if θ belongs to a separating interval, then $\hat{\theta}(\theta) = \theta$. Fix an $\varepsilon > 0$, by the continuity of $J(\theta, \hat{\theta}, z)$, there exists a $\delta_1 > 0$ such that for each θ' with $|\theta' - \theta| < \delta_1$, we have

$$|J(\theta, \hat{\theta}(\theta'), z^u(\theta')) - J(\theta', \hat{\theta}(\theta'), z^u(\theta'))| < \varepsilon.$$

In addition, the optimality of $z^u(\theta)$ implies that

$$J(\theta, \hat{\theta}(\theta), z^u(\theta)) \geq J(\theta, \hat{\theta}(\theta'), z^u(\theta')) > J(\theta', \hat{\theta}(\theta'), z^u(\theta')) - \varepsilon.$$

On the other hand, there exists a $\delta_2 > 0$ such that for each θ' with $|\theta' - \theta| < \delta_2$, we have

$$|J(\theta', \hat{\theta}(\theta), z^u(\theta)) - J(\theta, \hat{\theta}(\theta), z^u(\theta))| < \varepsilon.$$

Similarly, the optimality of $z^u(\theta')$ implies that

$$J(\theta', \hat{\theta}(\theta'), z^u(\theta')) \geq J(\theta', \hat{\theta}(\theta), z^u(\theta)) > J(\theta, \hat{\theta}(\theta), z^u(\theta)) - \varepsilon.$$

Together, we have that for each θ' with $|\theta' - \theta| < \min\{\delta_1, \delta_2\}$,

$$J(\theta, \hat{\theta}(\theta), z^u(\theta)) - \varepsilon < J(\theta', \hat{\theta}(\theta'), z^u(\theta')) < J(\theta, \hat{\theta}(\theta), z^u(\theta)) + \varepsilon.$$

Note that $J(\theta', \hat{\theta}(\theta'), z^u(\theta')) = J^u(z^u(\theta'), \theta')$. Thus, the lemma is proven. □

Henceforth, we focus on the equilibrium in which $z^u(\theta)$ is piecewise continuous. Without loss of generality, assume that $z^u(\theta)$ is right-continuous at each point of discontinuity and is continuous at the end points. Thus, all equilibria can be divided into two groups: in the first, $\bar{\theta}$ belongs to a separating interval, including a separating equilibrium with $z^u(\theta)$ increasing on $[\theta_0^u, \bar{\theta}]$; in the second, $\bar{\theta}$ belongs to a pooling interval, including a pooling equilibrium with $z^u(\theta) \equiv \tilde{z}$ on $[\theta_0^u, \bar{\theta}]$ for some $\tilde{z} > 0$. Whereas it is complex and unnecessary for the paper to fully characterize these equilibria, we provide a useful result for such equilibria as follows.

Lemma 5. *Suppose in some equilibrium, $\bar{\theta}$ belongs to a separating interval. Let $[\theta_1, \bar{\theta}]$ be the maximal separating interval incorporating $\bar{\theta}$, and let $[\theta_2, \theta_1)$ be the maximal interval adjacent to $[\theta_1, \bar{\theta}]$ such that $z^u(\theta)$ is continuous. If the equilibrium is not the seller-optimal separating equilibrium, then $z^u(\theta)$ is constant on $[\theta_2, \theta_1)$, jumps discontinuously at θ_1, and is continuous and satisfies (4.3) on $[\theta_1, \bar{\theta}]$. Moreover, $z^u(\theta) > z^*(\theta)$ on $[\theta_1, \bar{\theta}]$.*

Proof. There are two possibilities. First, if $\theta_1 = \theta_0^u$, then we have a separating equilibrium, and thus, $[\theta_2, \theta_1) = [\underline{\theta}, \theta_0^u)$ where $z^u(\theta) \equiv 0$. By Mailath and von Thadden (2013, Lemma G), there exists at most one point of discontinuity θ' on $(\theta_1, \bar{\theta})$ such that $\lim_{\theta \uparrow \theta'} z^u(\theta) < z^*(\theta') < \lim_{\theta \downarrow \theta'} z^u(\theta)$. Then, by Mailath and von Thadden (2013, Lemma A), $z^u(\theta)$ satisfies (4.3) in both the left and right neighborhoods of θ'. But by the proof of Theorem 3, $z^u(\theta) > z^*(\theta)$ in both neighborhoods, a contradiction, and thus, $z^u(\theta)$ is continuous on $[\theta_1, \bar{\theta}]$. It follows from Mailath and von Thadden (2013, Lemmas A and B) and the proof of Theorem 3 that $z^u(\theta)$ satisfies (4.3) and $z^u(\theta) > z^*(\theta)$ on $(\theta_1, \bar{\theta}]$. Then, we claim that $\theta_1 > \theta_0^o$, as the equilibrium is not the seller-optimal separating equilibrium. To see this, first note that if $\theta_1 = \theta_0^o$, then $z^u(\theta_1) = z^*(\theta_1)$. Specifically, if $\theta_1 = \theta_0^o = \underline{\theta}$, then from the previous argument, $z^u(\underline{\theta}) \geq z^*(\underline{\theta})$. Suppose $z^u(\underline{\theta}) > z^*(\underline{\theta})$, then the school can profit by assigning $z^*(\underline{\theta})$ to a neighborhood of $\underline{\theta}$, because the employers cannot punish such a deviation by having a worse belief than $\underline{\theta}$ and $J^o(z, \theta)$ is continuous and strictly concave in z. Thus, we have $z^u(\underline{\theta}) = z^*(\underline{\theta})$. If $\theta_1 = \theta_0^o > \underline{\theta}$, then $z^u(\theta_1) = z^o(\theta_1) = z^*(\theta_1) = 0$. It follows that if in some separating equilibrium, $\theta_0^u = \theta_0^o$, then it must be the seller-optimal separating equilibrium. This in turn implies that $\theta_1 \neq \theta_0^o$ in the current equilibrium. Note that $J^u(z^u(\theta), \theta) = J^o(z^u(\theta), \theta)$ for $\theta \geq \theta_1$. Thus, if $\theta_1 < \theta_0^o$, then $0 < J^u(z^u(\theta_0^o), \theta_0^o) \leq J^o(z^*(\theta_0^o), \theta_0^o) = 0$, a contradiction. In summary, we have $\theta_1 > \theta_0^o$. It follows that $z^u(\theta_1) \geq z^*(\theta_1) > 0$ and $J^o(z^u(\theta_1), \theta_1) = 0$. This implies that $z^u(\theta_1)$ is the maximal root of $J^o(z, \theta_1) = 0$. Thus, $z^u(\theta)$ jumps discontinuously at θ_1 with $z^u(\theta_1) > z^*(\theta_1)$. Then, by Mailath and von Thadden (2013, Lemma A), $z^u(\theta)$ also satisfies (4.3) at θ_1.

Second, if $\theta_1 > \theta_0^u$, then $z^u(\theta)$ is discontinuous at θ_1. Suppose not, then $z^u(\theta)$ is constant in the left neighborhood of θ_1. It follows that the wage $W^u(z)$ has an upward jump at $z^u(\theta_1)$. Then, the school can profit by assigning $z^u(\theta')$ to a neighborhood of θ_1 for some $\theta' > \theta_1$, a contradiction. Moreover, it follows that $z^u(\theta)$ is either separating or pooling on $[\theta_2, \theta_1)$ since $z^u(\theta)$ is continuous there. But since $z^u(\theta)$ is discontinuous at θ_1 and is separating on $[\theta_1, \bar{\theta}]$, the above paragraph implies that $[\theta_2, \theta_1)$ is a pooling interval and that $z^u(\theta)$ is continuous and $z^u(\theta) \geq z^*(\theta)$ on $[\theta_1, \bar{\theta}]$ with strict inequality for $\theta > \theta_1$. Suppose $z^u(\theta_1) = z^*(\theta_1)$, then we have $J^u(z^u(\theta_1), \theta_1) = J^o(z^*(\theta_1), \theta_1) > \lim_{\theta \uparrow \theta_1} J^u(z^u(\theta), \theta)$ because $z^u(\theta)$ is discontinuous at θ_1 and $z^*(\theta_1)$ is the unique maximizer of $J^o(z, \theta_1)$. But this contradicts Lemma 4 which states that $J^u(z^u(\theta),\theta)$ is continuous at θ_1. Thus, $z^u(\theta_1) > z^*(\theta_1)$. It follows from the above paragraph that $z^u(\theta)$ also satisfies (4.3) at θ_1. Thus, the lemma is proven. □

The proof of Lemma 5 implies that a continuous equilibrium where $z^u(\theta)$ is continuous on $[\theta_0^u, \bar{\theta}]$ is either separating or pooling. Moreover, it leads to the following corollary.

Corollary 3. *In each separating equilibrium of the unobserved case, $z^u(\theta)$ is continuous on $[\theta_0^u, \bar{\theta}]$, and satisfies (4.3) on $(\theta_0^u, \bar{\theta}]$, with $\theta_0^u \geq \theta_0^o$. If $\theta_0^u = \theta_0^o$, then $z^u(\theta_0^u) = z^*(\theta_0^u) \geq z^o(\theta_0^u)$. If $\theta_0^u > \theta_0^o$, then $z^u(\theta_0^u)$ is the maximal root of $J^o(z, \theta_0^u) = 0$, and (4.3) also holds at θ_0^u.*

Corollary 3 implies that given an equilibrium cutoff type, his education level is uniquely determined. Thus, we can characterize the set of equilibrium initial points by characterizing the set of cutoff types. Note that the lower bound of θ_0^u is θ_0^o. Moreover, we can determine the upper bound of θ_0^u by assuming the worst off-path belief. Specifically, denote by $\bar{\theta}_0^u$ the upper bound of θ_0^u, which is the maximal root of

$$\max_{z\geq 0}\{Q(z, \underline{\theta}) - G(z, \theta)\} = 0$$

if it exists or $\underline{\theta}$ otherwise. Therefore, if $\bar{\theta}_0^u = \underline{\theta}$, then the cutoff type and thus the separating equilibrium outcome is unique. In contrast, if $\bar{\theta}_0^u > \underline{\theta}$, then any type between θ_0^o and $\bar{\theta}_0^u$ can be an equilibrium cutoff type for some proper off-path belief. Thus, the set of cutoff types is given by $[\theta_0^o, \bar{\theta}_0^u]$. It follows from Corollary 3 that $z^u(\theta_0^u)$ is continuous and increasing in θ_0^u. Figure 4 illustrates the set of education functions of separating equilibria. As depicted, each education function satisfies that $z^u(\theta) \geq z^o(\theta)$ on $[\theta_0^u, \bar{\theta}]$ with strict inequality for $\theta > \theta_0^u$.

Now, we consider pooling equilibria to fully characterize the set of continuous equilibria. For each θ, define a function of z on $\mathbb{R}_+$ as follows:

$$\Delta(z, \theta) := \mathbb{E}\{Q(z, \theta) | \theta \in [\underline{\theta}, \bar{\theta}]\} - G(z, \theta) - \max_{y \geq 0}\{Q(y, \underline{\theta}) - G(y, \theta)\}.$$

Note that $\Delta(z, \theta)$ is the net gain in virtual surplus of type θ if in equilibrium all types pool at education level z, compared to type θ's optimal deviation under the worst off-path belief. Because $Q_{zz} \leq 0$ and $G_{zz} > 0$, $\Delta(z, \theta)$ is strictly concave in z for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Moreover, it is clear that $\Delta(0, \theta) \leq 0$ and that if $\Delta(0, \theta) < 0$, then $\Delta(z, \theta) > 0$ for some $z > 0$. Thus, for each θ, $\Delta(z, \theta)$ has a root and at most two roots on $\mathbb{R}_+$. Let $\underline{x}(\theta)$ and $\bar{x}(\theta)$ be the minimal and maximal roots of $\Delta(z, \theta) = 0$, respectively, with the possibility that $\underline{x}(\theta) = \bar{x}(\theta) = 0$.

The next proposition provides a necessary and sufficient condition for the existence of pooling equilibrium when $z^*(\underline{\theta}) > 0$.

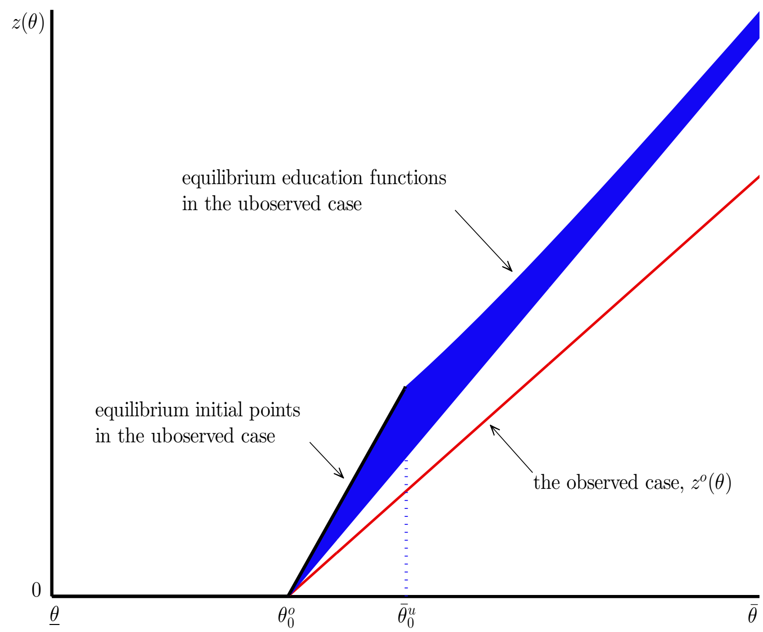


Figure 4. **The Set of Separating Equilibria.** This figure illustrates the set of separating equilibria of the unobserved case given that $\theta_0^o > \underline{\theta}$. The shaded area depicts the set of equilibrium education functions. This region is uniformly above the equilibrium education function of the observed case $z^o(\theta)$. The bold line is the set of equilibrium initial points with the cutoff type ranging from θ_0^o to $\bar{\theta}_0^u$. Each point uniquely determines an equilibrium education function $z^u(\theta)$ and thus an equilibrium outcome. This figure considers the same numerical example as Figure 2 such that the set of the initial points is $\{(\theta, z)|z(\theta) = 3\theta - 1, \frac{1}{3} \leq \theta \leq \frac{1}{2}\}$.

Proposition 5. *Suppose $z^*(\underline{\theta}) > 0$, then a pooling equilibrium exists in the unobserved case if and only if $\bar{x}(\underline{\theta}) \geq \underline{x}(\bar{\theta})$.*

Proof. First, we show that a pooling equilibrium exists if $\bar{x}(\underline{\theta}) > \underline{x}(\bar{\theta})$. For each θ, define

$$y(\theta) := \underset{z \geq 0}{\operatorname{argmax}}\, Q(z, \underline{\theta}) - G(z, \theta).$$

It is clear that $y(\theta)$ is nondecreasing on $[\underline{\theta}, \bar{\theta}]$. By the envelope theorem, for any $z > 0$,

$$\Delta_\theta(z, \theta) = G_\theta(y(\theta), \theta) - G_\theta(z, \theta).$$

Because $G_{z\theta} < 0$, $\Delta_\theta(z, \theta) > 0$ if and only if $y(\theta) < z$. It follows that for any fixed $z > 0$, $\Delta(z, \theta)$ is strictly quasiconcave on $[\underline{\theta}, \bar{\theta}]$. Assume that the labor market regards any off-path education level as chosen by the lowest type. Therefore, a pooling equilibrium exists if there exists some $\tilde{z} > 0$ such that $\Delta(\tilde{z}, \theta) \geq 0$ on $[\underline{\theta}, \bar{\theta}]$. This reduces to that $\Delta(\tilde{z}, \underline{\theta}), \Delta(\tilde{z}, \bar{\theta}) \geq 0$, as $\Delta(\tilde{z}, \cdot)$ is strictly quasiconcave. We consider two cases. First, if $y(\underline{\theta}) > 0$, then $y(\bar{\theta}) > y(\underline{\theta})$; thus, $\Delta(y(\underline{\theta}), \underline{\theta}), \Delta(y(\bar{\theta}), \bar{\theta}) > 0$. It follows that $0 < \underline{x}(\underline{\theta}) < \bar{x}(\bar{\theta})$. Because $\bar{x}(\underline{\theta}) > \underline{x}(\bar{\theta})$, there exists a $\tilde{z}$ such that $\max\left\{\underline{x}(\underline{\theta}), \underline{x}(\bar{\theta})\right\} \leq \tilde{z} \leq \min\left\{\bar{x}(\underline{\theta}), \bar{x}(\bar{\theta})\right\}$, and thus, $\Delta(\tilde{z}, \underline{\theta}), \Delta(\tilde{z}, \bar{\theta}) \geq 0$.

Second, if $y(\underline{\theta}) = 0$, then $y(\underline{\theta}) = \underline{x}(\underline{\theta})$. Because $\bar{x}(\underline{\theta}) > 0$, $\Delta(z, \underline{\theta}) > 0$ on $(0, \bar{x}(\underline{\theta}))$. Therefore, if $y(\bar{\theta}) > 0$, then $0 < \underline{x}(\bar{\theta}) < \bar{x}(\bar{\theta})$, and thus, for any $\tilde{z}$ such that $\underline{x}(\bar{\theta}) \le \tilde{z} \le \min\{\bar{x}(\underline{\theta}), \bar{x}(\bar{\theta})\}$, $\Delta(\tilde{z}, \underline{\theta}), \Delta(\tilde{z}, \bar{\theta}) \ge 0$; if $y(\bar{\theta}) = 0$, then $\Delta(z, \bar{\theta}) \ge \Delta(z, \underline{\theta})$ because $G_\theta < 0$, and thus, for any $\tilde{z} \in (0, \bar{x}(\underline{\theta})]$, $\Delta(\tilde{z}, \underline{\theta}), \Delta(\tilde{z}, \bar{\theta}) \ge 0$. In summary, a pooling equilibrium exists if $\bar{x}(\underline{\theta}) > \underline{x}(\bar{\theta})$. The proof can be extended to $\bar{x}(\underline{\theta}) \ge \underline{x}(\bar{\theta})$ if $z^*(\underline{\theta}) > 0$ since then $y(\underline{\theta}) = z^*(\underline{\theta}) > 0$.

Then, we consider the converse. Suppose a pooling equilibrium exists. Since $J^o(z, \underline{\theta})$ is strictly concave in z and $z^*(\underline{\theta}) > 0$, for any $z \in (0, z^*(\underline{\theta}))$, $J^u(z, \underline{\theta}) \ge J^o(z, \underline{\theta}) > 0$. It follows that $\theta_0^u = \underline{\theta}$. In addition, $y(\bar{\theta}) > y(\underline{\theta}) = z^*(\underline{\theta}) > 0$, thus both $\Delta(z, \underline{\theta})$ and $\Delta(z, \bar{\theta})$ have two zeroes. Let $\tilde{z} > 0$ be the equilibrium education level. We claim that $\tilde{z} \le \bar{x}(\underline{\theta})$. Suppose not, then $\Delta(\tilde{z}, \underline{\theta}) < 0$; thus, the school can profit by decreasing $z(\theta)$ in some neighborhood of $\underline{\theta}$, with the monotonicity still holding. But note that $\Delta(z, \theta)$ assumes the worst off-path belief, meaning that any different off-path belief makes such a deviation weakly more tempting, a contradiction. Similarly, suppose $\tilde{z} < \underline{x}(\bar{\theta})$, then $\Delta(\tilde{z}, \bar{\theta}) < 0$; thus, the school can profit by increasing $z(\theta)$ in some neighborhood of $\bar{\theta}$, with the monotonicity still holding, irrespective of the off-path belief, a contradiction. In summary, we must have $\underline{x}(\bar{\theta}) \le \tilde{z} \le \bar{x}(\underline{\theta})$. That is, if a pooling equilibrium exists, then $\bar{x}(\underline{\theta}) \ge \underline{x}(\bar{\theta})$. Thus, the proposition is proven. □

Note that when $z^*(\underline{\theta}) > 0$, $\bar{\theta}_0^u = \underline{\theta} = \theta_0^o$, thus there exists a unique separating equilibrium outcome such that $(\theta_0^u, z^u(\theta_0^u)) = (\underline{\theta}, z^*(\underline{\theta}))$. If further $\bar{x}(\underline{\theta}) < \underline{x}(\bar{\theta})$, then there does not exist any pooling equilibrium. Therefore, the unique continuous equilibrium is the seller-optimal separating equilibrium in which $(\theta_0^u, z^u(\theta_0^u)) = (\underline{\theta}, z^*(\underline{\theta}))$. To summarize,

Corollary 4. *If $z^*(\underline{\theta}) > 0$ and $\bar{x}(\underline{\theta}) < \underline{x}(\bar{\theta})$, then there exists a unique continuous equilibrium in the unobserved case, that is, the seller-optimal separating equilibrium.*

Example. To illustrate, we provide an example in which there exists a unique continuous equilibrium. Assume that $Q(z, \theta) = \theta z + 2z$, $C(z, \theta) = 1.4z^2 + 2.8z - 1.4\theta z$, and $\theta \sim U[1, 2]$. It follows that $G(z, \theta) = 1.4z^2 + 5.6z - 2.8\theta z$ and $\mathbb{E}[Q(z, \theta)|1 \le \theta \le 2] = 3.5z$. From basic calculation, $z^*(1) = y(1) = 1/14$ and $y(2) = 15/14$. Thus,

$$Q(y(1), 1) - G(y(1), 1) = \frac{1}{140} \quad \text{and} \quad Q(y(2), 1) - G(y(2), 2) = \frac{45}{28}.$$

Substituting, we have

$$\Delta(z, 1) = -1.4z^2 + 0.7z - \frac{1}{140} \quad \text{and} \quad \Delta(z, 2) = -1.4z^2 + 3.5z - \frac{45}{28}.$$

It follows that $\bar{x}(1) \approx 0.49$ and $\underline{x}(2) \approx 0.61$. Thus, from Corollary 4, the unique continuous equilibrium is the seller-optimal separating equilibrium such that $(\theta_0^u, z^u(\theta_0^u)) = (1, 1/14)$.

D Signaling intensity and welfare comparison

In this section, we investigate how the welfare of each case considered in the paper depends on the intensity of signaling. To make the concept of signaling intensity more concrete and for expositional convenience, we consider a parametric example below.

Parametric example. Assume that $Q(z,\theta) = \gamma\theta z + z$ with $\gamma > 0$, $C(z,\theta) = z^2 + z - \theta z$, and $\theta \sim U[0,1]$. Applying the results of the paper, we have $z^{fb}(\theta) = \frac{(\gamma+1)\theta}{2}$, $z^s(\theta) = \frac{(2\gamma+1)\theta}{2}$, $z^o(\theta) = \frac{(\gamma+2)\theta-1}{2}$ and $z^u(\theta) = (\gamma+1)\theta - \frac{\gamma+1}{\gamma+2}$. Therefore, $z^u(\theta) \geq z^{fb}(\theta)$ if and only if $\theta \geq \frac{2}{\gamma+2}$. It is heuristic to define the *intensity of signaling* as the ratio of the overinvested education in Spence's game, i.e., $z^s(\theta) - z^{fb}(\theta)$, to the first-best level $z^{fb}(\theta)$. Substituting the education functions, we have that for each $\theta > 0$,

$$\frac{z^s(\theta) - z^{fb}(\theta)}{z^{fb}(\theta)} = \frac{\gamma}{\gamma+1}.$$

Clearly, the intensity of signaling is increasing in γ. Intuitively, the larger γ is, the stronger complementarity between ability and education. In Spence's game, higher education signals higher ability. If ability complements education to a larger extent, the marginal benefit of education will be higher, thereby enhancing signaling through education.

Then, we investigate how signaling intensity affects signaling mitigating the screening distortion in the observed case. Analogously, we define the degree of screening distortion as the ratio of the underinvested education, i.e., $z^{fb}(\theta) - z^o(\theta)$, to the first-best level $z^{fb}(\theta)$. Substituting the education functions, we have that for each $\theta > 0$,

$$\frac{z^{fb}(\theta) - z^o(\theta)}{z^{fb}(\theta)} = \frac{1-\theta}{(\gamma+1)\theta}.$$

For each fixed $\theta \in (0,1)$, the degree of screening distortion is decreasing in γ. This means that the more intense signaling is, the more screening distortion is reduced.

In the unobserved case, however, the more intense signaling is, the more over-education. Note that the cutoff $\frac{2}{\gamma+2}$ is decreasing in γ. That is, the over-education region is increasing in the intensity of signaling. Subtracting the total surplus of the unobserved case from that of the observed case, we have

$$\int_{\underline{\theta}}^{\bar{\theta}} [S(z^o(\theta),\theta) - S(z^u(\theta),\theta)]\, dF(\theta) = \frac{\gamma(\gamma-1)(\gamma+1)^3}{12(\gamma+2)^3}.$$

Clearly, the RHS is positive if and only if $\gamma > 1$. That is, if signaling is sufficiently intense, then the observed case yields higher social welfare than the unobserved case.

Then, comparing welfare between the observed case and Spence's game, we have

$$\int_{\underline{\theta}}^{\bar{\theta}} [S(z^o(\theta),\theta) - S(z^s(\theta),\theta)]\, dF(\theta) = \frac{(\gamma^2+\gamma-1)(\gamma^2+3\gamma+1)}{12(\gamma+2)^2}.$$

Therefore, the observed case yields higher social welfare if and only if $\gamma > \frac{\sqrt{5}-1}{2}$. It can also be shown that the unobserved case yields higher social welfare than Spence's game if and only if γ is larger than some cutoff less than $\frac{\sqrt{5}-1}{2}$. In summary, as the intensity of signaling increases (i.e., as γ increases), the case that yields the highest social welfare will be Spence's game, the unobserved case, and the observed case, respectively.

E Applications of the model

To demonstrate model applicability, we present in a parallel manner to job market signaling an adapted version of the model for conspicuous consumption and advertising.

Conspicuous consumption. A luxury good retailer (*seller*) first chooses a price scheme $T : \mathbb{R}_+ \to \mathbb{R}_+$, which specifies the price for each level of quality z. Then, à la Bagwell and Bernheim (1996), a consumer (*buyer*) chooses a quality level to signal his privately known wealth (*type*) θ to a representative social contact (*receiver*). The social contact observes z. In the observed case, she also observes T; in the unobserved case, she does not observe T.

In the spirit of the classic work of Veblen (1899), the social contact rewards the consumer according to z. The reward $W(z)$ is given by the social contact's expected benefit $\mathbb{E}[Q(z,\theta)]$ from the consumer. The benefit function $Q(z,\theta)$ is increasing in both arguments: Q_z, $Q_\theta > 0$ if $z > 0$. This is because a social contact benefits more from establishing relationships with wealthier people and from interacting with people who consume higher quality goods (e.g., the good may be nonrivalrous). We also assume that $Q_{zz} \le 0$ and $Q(0,\theta) \equiv 0$.[14] Moreover, the consumer derives intrinsic utility $V(z,\theta)$ from the luxury good, which is increasing in quality z. We assume that the single-crossing property holds: $V_{z\theta} > 0$. That is, a wealthier individual derives higher marginal utility from a luxury good. For example, a consumer of yacht can voyage more often if he is richer, since he is better able to afford the fuel costs and maintenance fees. Thus, a type-θ consumer who chooses quality z has utility

$$u(z,\theta) \coloneqq W(z) + V(z,\theta) - T(z).$$

The retailer's profit equals the revenue $T(z)$ minus the cost $C(z)$, with C', $C'' > 0$ and $C(0) = 0$. The social surplus function is thus given by $S(z,\theta) \coloneqq Q(z,\theta) + V(z,\theta) - C(z)$.

[14]The second condition captures the idea that the consumer may need at least an entry-level luxury good to meet the social contact. For example, to join a yacht club, one usually has to own a yacht.

Advertising. A media company (*seller*) first chooses a price scheme $T : \mathbb{R}_+ \to \mathbb{R}_+$, which specifies the price for each level of advertising z. Then, à la Milgrom and Roberts (1986), a producer of a new product (*buyer*) chooses an advertising level to signal the privately known product quality (*type*) θ to consumers. The consumers observe z. In the observed case, they also observe T; in the unobserved case, they do not observe T.

The producer's total revenue consists of two parts: the purchase in the introductory stage and the repeat purchase in the post-introductory stage. In the introductory stage, $D(z) \geq 0$ consumers become aware of the product, and each purchases one unit at a price equal to the expected quality $\mathbb{E}[\theta|z]$. The demand function $D(z)$ is increasing, as more advertising leads to higher consumer awareness. We also assume that $D_{zz} \leq 0$ and $D(0) = 0$. Then, in the post-introductory stage, the product's actual quality θ is revealed, and thus, the consumers are willing to purchase the good again at a price equal to θ. We assume that the consumers who were unaware of the product do not purchase the good in the post-introductory stage. Thus, the producer's total revenue equals $(\mathbb{E}[\theta|z] + \theta)D(z)$, and its net payoff is given by

$$u(z, \theta) \coloneqq (\mathbb{E}[\theta|z] + \theta)D(z) - T(z).$$

Note that the single-crossing property holds: $u_{z\theta} > 0$. This is due to the complementarity between advertising and quality, i.e., the marginal revenue of the introductory advertising is higher if the product is of higher quality, thereby allowing the producer to charge a higher price in the post-introductory stage. The media company's profit equals the revenue $T(z)$ minus the production cost $C(z)$, with C', $C'' > 0$ and $C(0) = 0$. The social surplus function is thus given by $S(z, \theta) \coloneqq 2\theta D(z) - C(z)$.

Then, applying the main results, we have that the demand for a luxury good (advertising) is more elastic if the social contact (consumers) cannot see the price scheme than otherwise. It follows that the retailer of luxury good charges lower prices and the consumer chooses a higher quality of the good when the social contact cannot observe the price scheme than they would otherwise. Similarly, the media company charges lower prices, and the producer chooses a higher advertising level, thereby obtaining a larger market share when consumers cannot observe the price scheme than they would otherwise. The results also suggest that price transparency benefits the seller of a luxury good (advertising) but harms the buyer. In addition, promoting competition in these markets might not be socially beneficial.

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